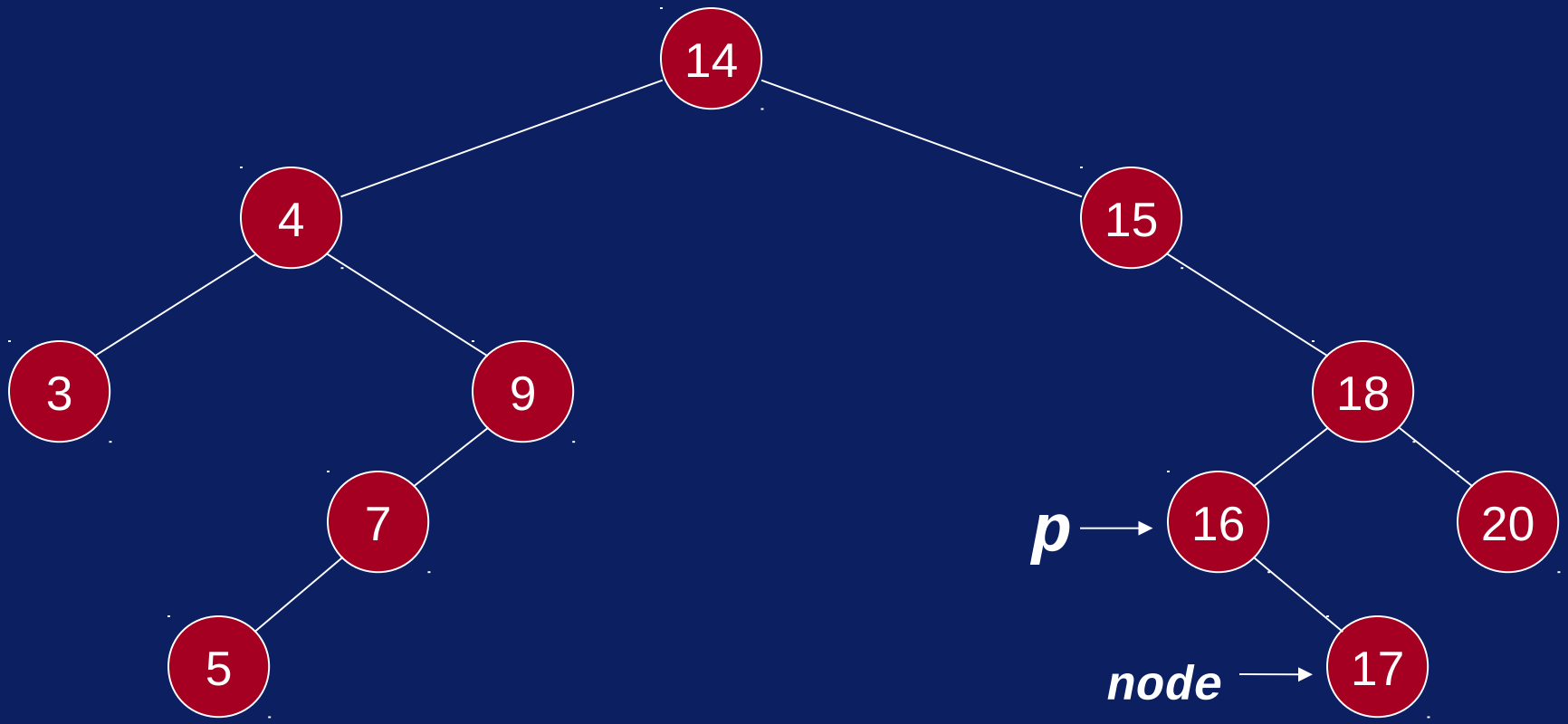
Lecture No.12
Binary Tree Traversal

CC-213 Data Structures
Department of Computer Science
University of the Punjab

Slides modified very slightly from the late Dr. Sohail Aslam's lectures at VU

Trace of insert



17, 9, 14, 5

```
p->setRight( node );
```

Cost of Search

- Given that a binary tree is level *d* deep. How long does it take to find out whether a number is already present?
- Consider the insert(17) in the example tree.
- Each time around the while loop, we did one comparison.
- After the comparison, we moved a level *down*.

Cost of Search

- With the binary tree in place, we can write a routine *find(x)* that returns true if the number *x* is present in the tree, false otherwise.
- How many comparison are needed to find out if *x* is present in the tree?
- We do one comparison at each level of the tree until either *x* is found or q becomes NULL.

Cost of Search

- If the binary tree is built out of *n* numbers, how many comparisons are needed to find out if a number *x* is in the tree?
- Recall that the depth of the complete binary tree built using '*n*' nodes will be $log_2(n+1) - 1$.
- For example, for n=100,000, $\log_2(100001)$ is less than 20; the tree would be 20 levels deep.

Cost of Search

- If the tree is complete binary or nearly complete, searching through 100,000 numbers will require a maximum of 20 comparisons.
- Or in general, approximately $log_2(n)$.
- Compare this with a linked list of 100,000 numbers. The comparisons required could be a maximum of n.

Binary Search Tree

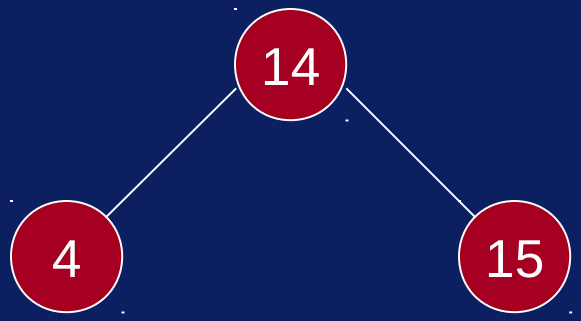
- A binary tree with the property that items in the left subtree are smaller than the root and items are larger or equal in the right subtree is called a *binary search tree* (BST).
- The tree we built for searching for duplicate numbers was a binary search tree.
- BST and its variations play an important role in searching algorithms.

Traversing a Binary Tree

- Suppose we have a binary tree, ordered (BST) or unordered.
- We want to print all the values stored in the nodes of the tree.
- In what order should we print them?

Traversing a Binary Tree

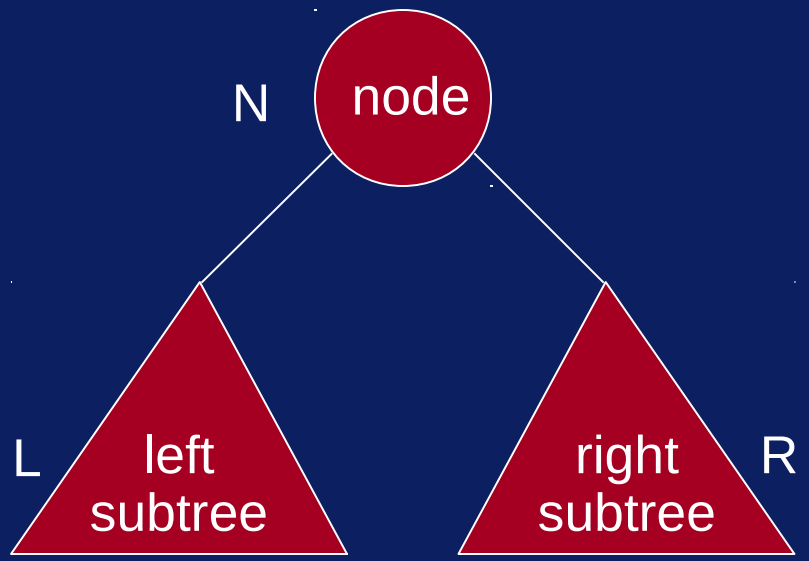
- Ways to print a 3 node tree:



(4, 14, 15), (4,15,14)
(14,4,15), (14,15,4)
(15,4,14), (15,14,4)

Traversing a Binary Tree

- In case of the general binary tree:



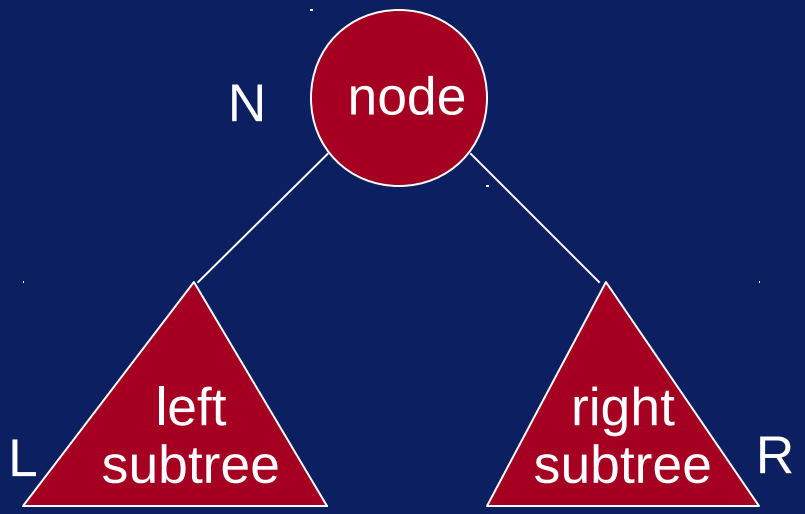
(L,N,R), (L,R,N)

(N,L,R), (N,R,L)

(R,L,N), (R,N,L)

Traversing a Binary Tree

- Three common ways



Preorder: (N,L,R)

Inorder: (L,N,R)

Postorder: (L,R,N)

Traversing a Binary Tree

```
void preorder(TreeNode<int>* treeNode)
{
    if( treeNode != NULL )
    {
        cout << *(treeNode->getInfo())<<"
  ";
        preorder(treeNode->getLeft());
        preorder(treeNode->getRight());
    }
}
```

Traversing a Binary Tree

```
void inorder(TreeNode<int>* treeNode)
{
    if( treeNode != NULL )
    {
        inorder(treeNode->getLeft());
        cout << *(treeNode->getInfo())<<"
  ";
        inorder(treeNode->getRight());
    }
}
```

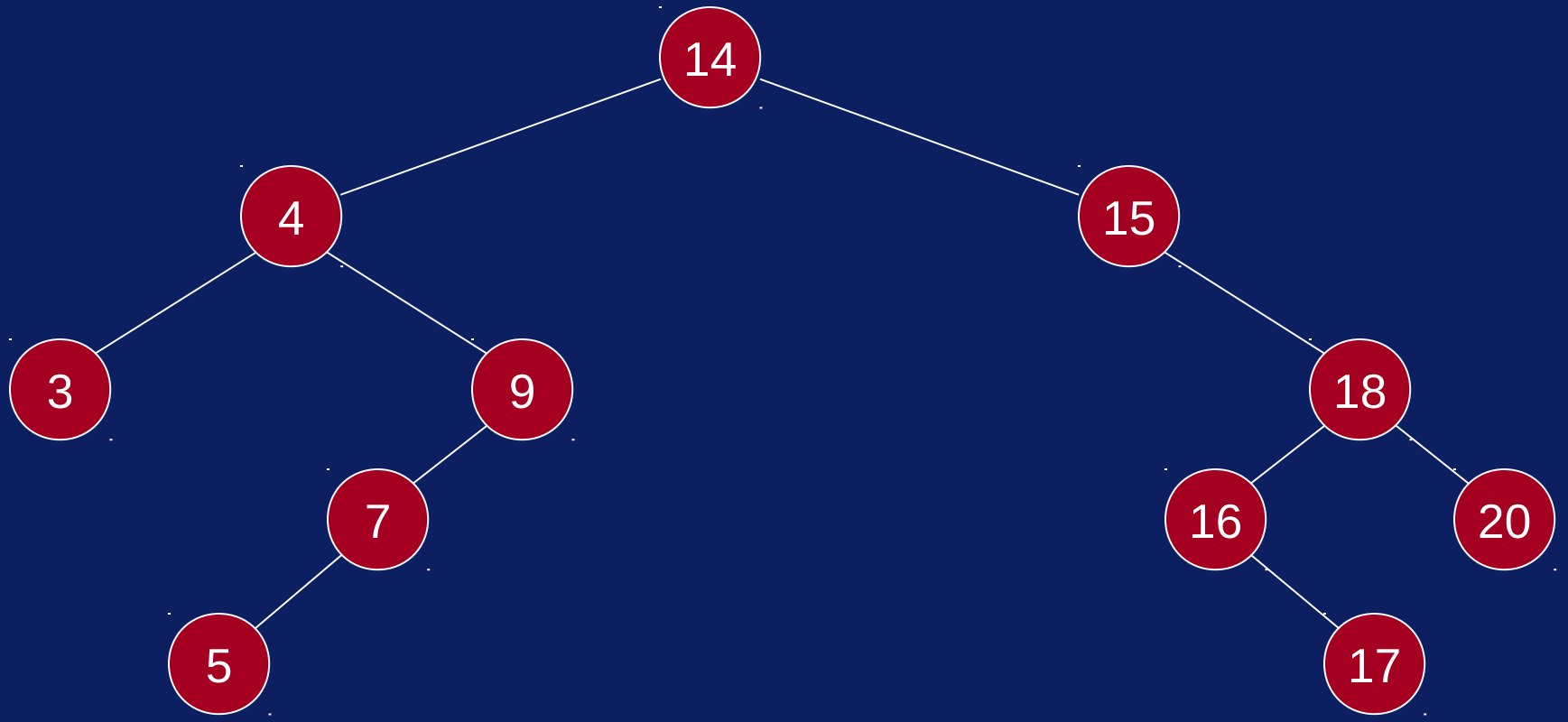
Traversing a Binary Tree

```
void postorder(TreeNode<int>* treeNode)
{
    if( treeNode != NULL )
    {
        postorder(treeNode->getLeft());
        postorder(treeNode->getRight());
        cout << *(treeNode->getInfo())<<"
  ";
    }
}
```

Traversing a Binary Tree

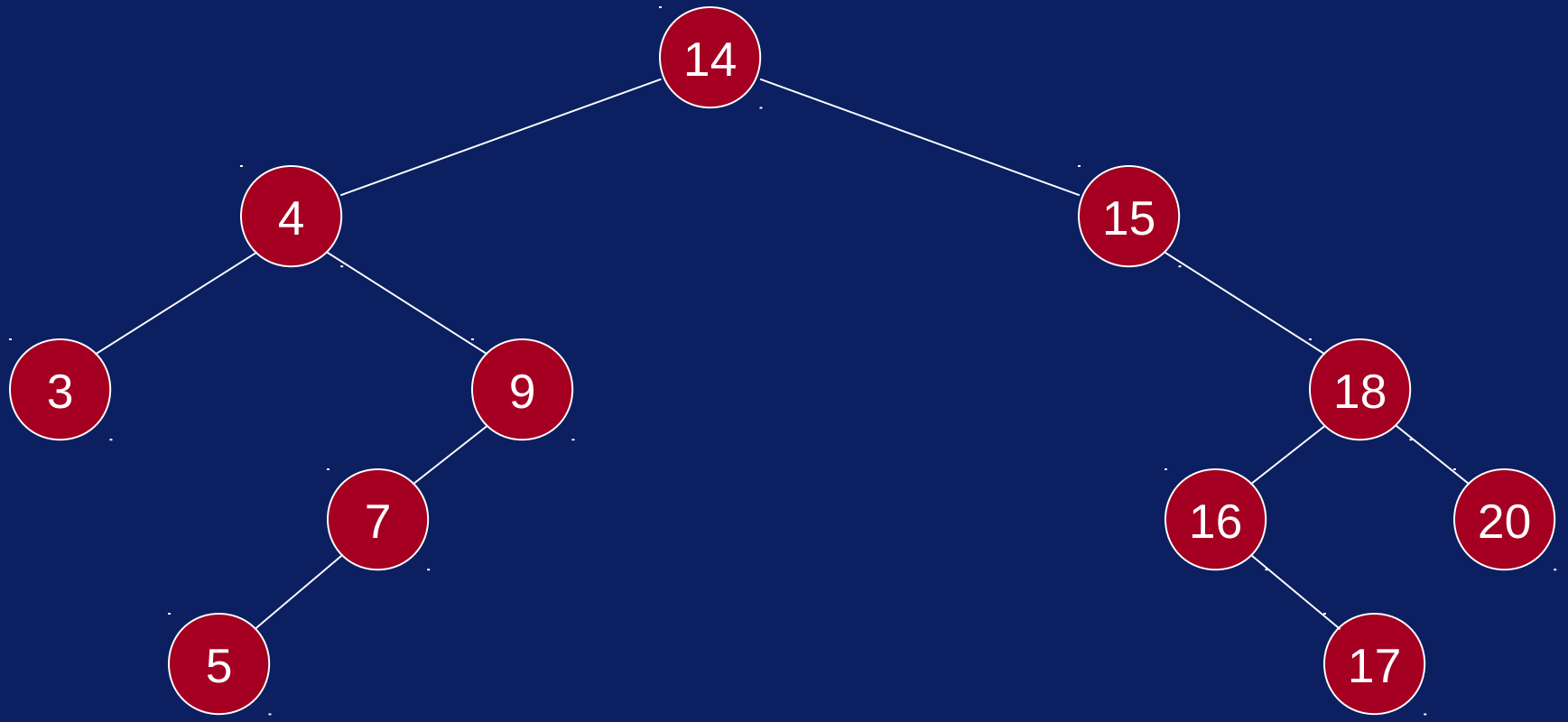
```
cout << "inorder: ";   preorder( root);
cout << "inorder: ";   inorder( root );
cout << "postorder: "; postorder( root );
```

Traversing a Binary Tree



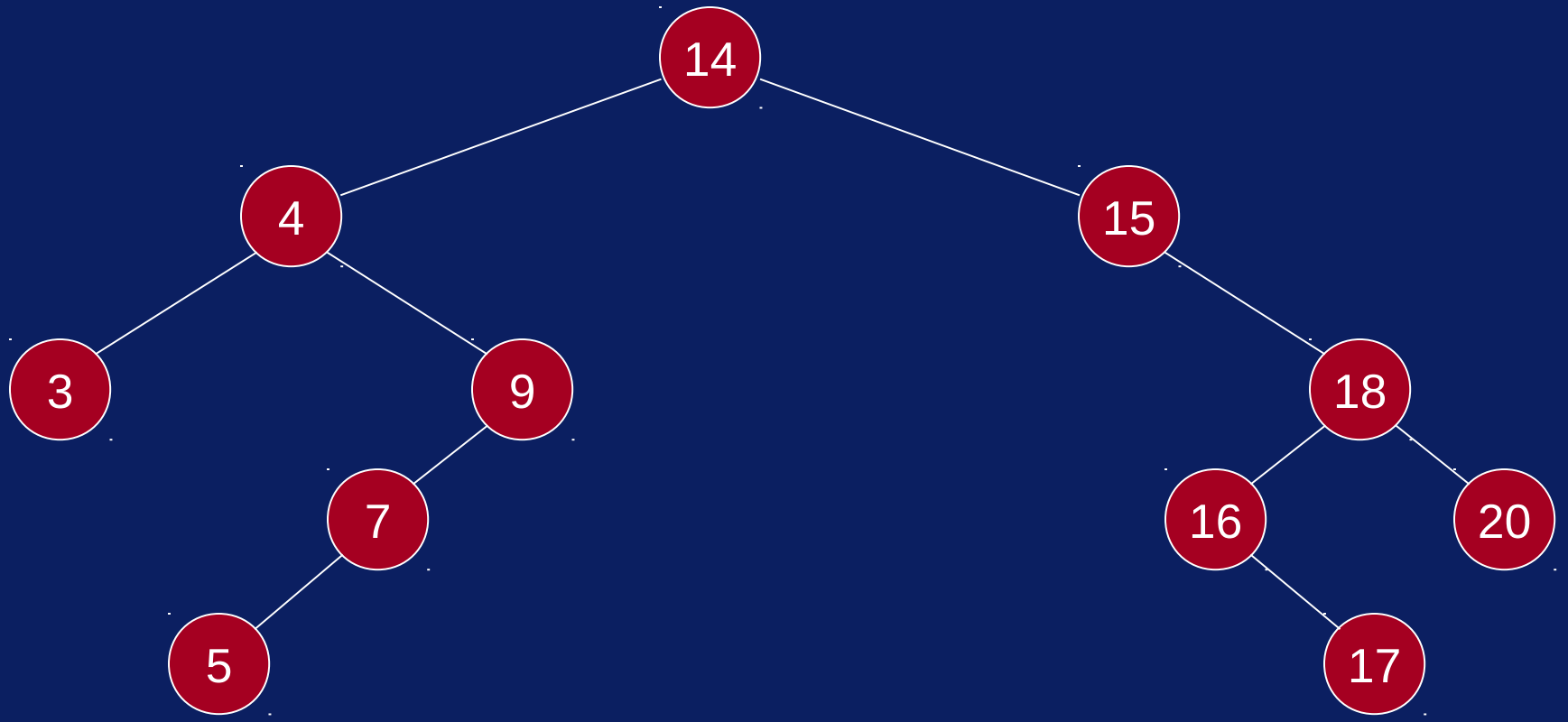
Preorder: 14 4 3 9 7 5 15 18 16 17 20

Traversing a Binary Tree



Inorder: 3 4 5 7 9 14 15 16 17 18 20

Traversing a Binary Tree



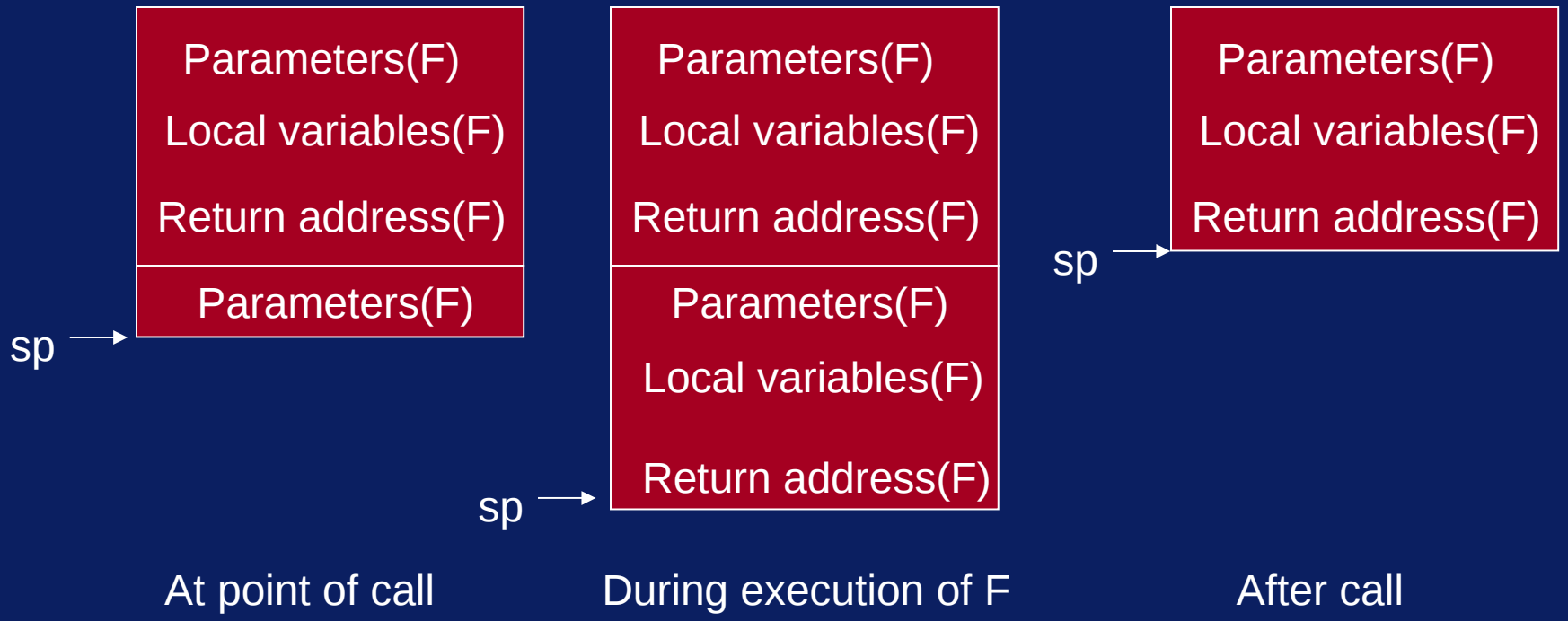
Postorder: 3 5 7 9 4 17 16 20 18 15 14

Recursive Call

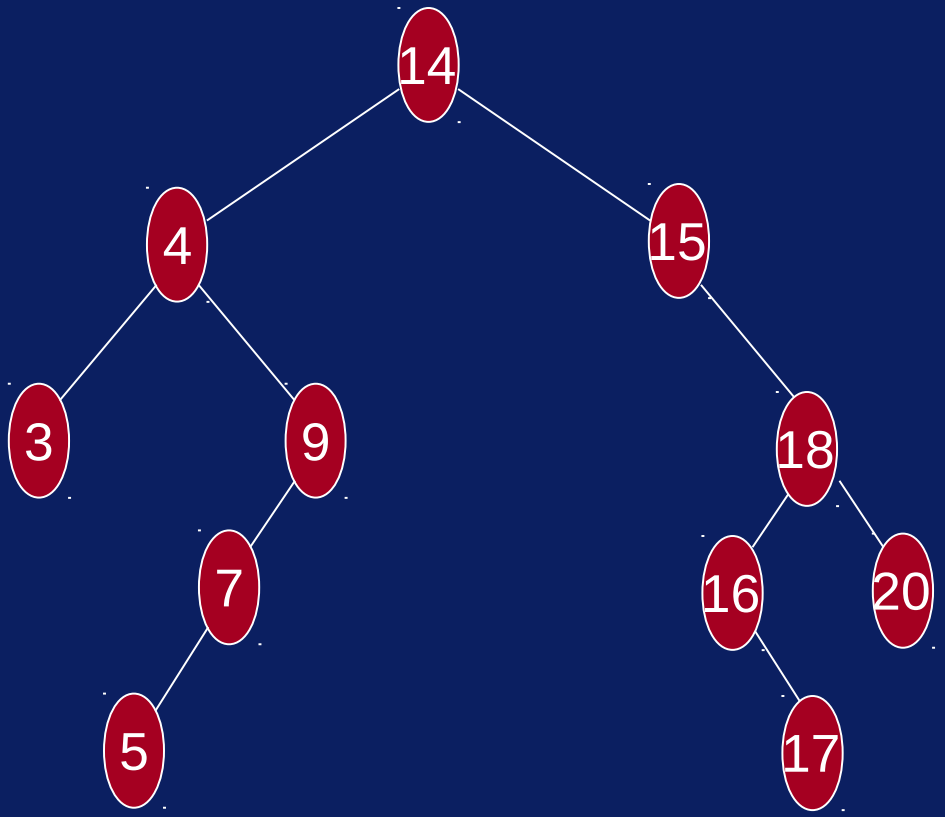
- Recall that a stack is used during function calls.
- The caller function places the arguments on the stack and passes control to the called function.
- Local variables are allocated storage on the call stack.
- Calling a function itself makes no difference as far as the call stack is concerned.

Stack Layout during a call

- Here is stack layout when function F calls function F (recursively):

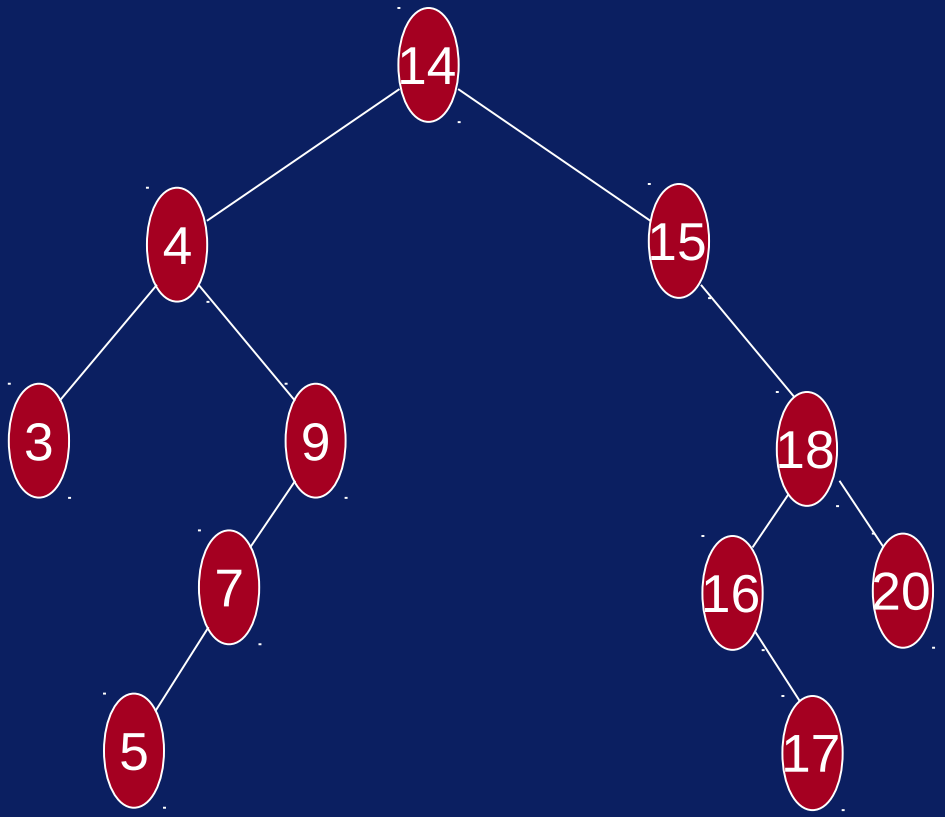


Recursion: preorder



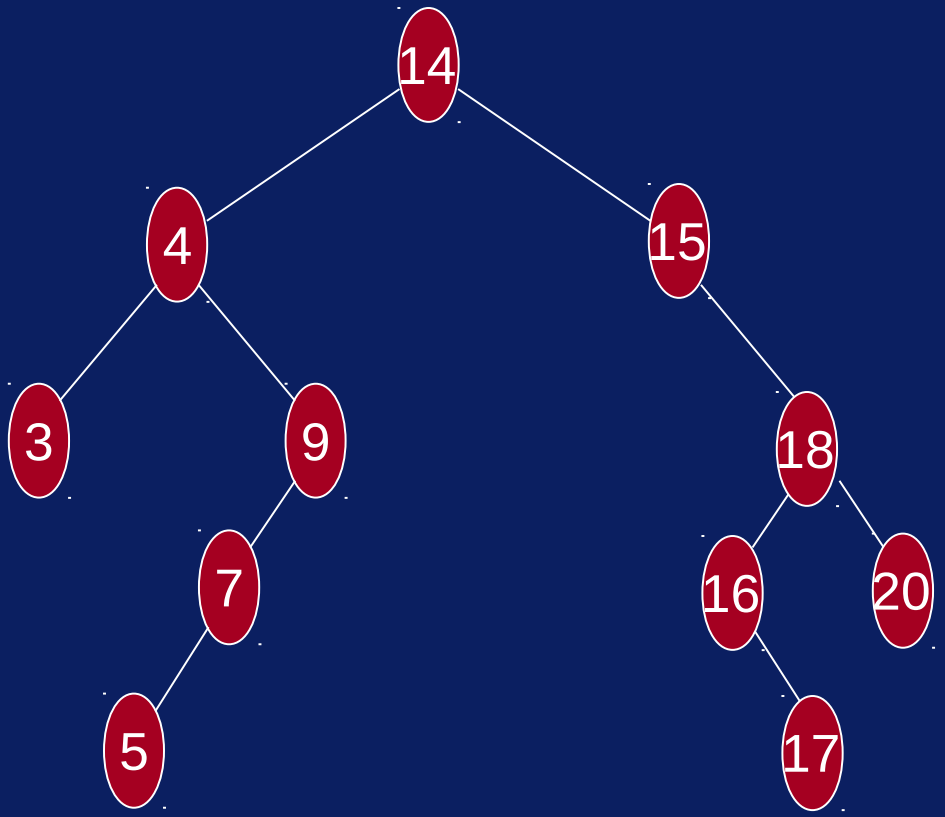

```
preorder(14)
14
..preorder(4)
4
....preorder(3)
3
......preorder(null)
......preorder(null)
....preorder(9)
9
......preorder(7)
7
........preorder(5)
5
..........preorder(null)
..........preorder(null)
........preorder(null)
......preorder(null)
```

Recursion: preorder



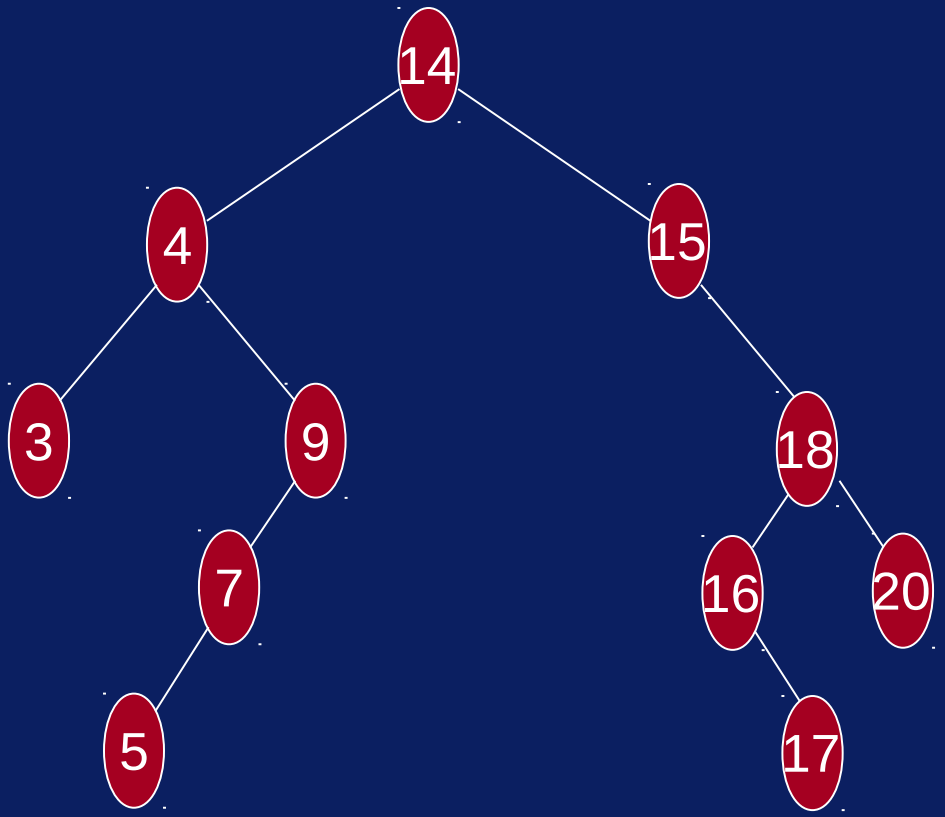

```
..preorder(15)
15
....preorder(null)
....preorder(18)
18
......preorder(16)
16
........preorder(null)
........preorder(17)
17
..........preorder(null)
..........preorder(null)
......preorder(20)
20
........preorder(null)
........preorder(null)
```

Recursion: inorder




```
inorder(14)
..inorder(4)
....inorder(3)
......inorder(null)
3
......inorder(null)
4
....inorder(9)
......inorder(7)
........inorder(5)
..........inorder(null)
5
..........inorder(null)
7
........inorder(null)
9
......inorder(null)
14
```

Recursion: inorder




```
..inorder(15)
....inorder(null)
15
....inorder(18)
......inorder(16)
........inorder(null)
16
........inorder(17)
..........inorder(null)
17
..........inorder(null)
18
......inorder(20)
........inorder(null)
20
........inorder(null)
```

Non Recursive Traversal

- We can implement non-recursive versions of the preorder, inorder and postorder traversal by using an explicit stack.
- The stack will be used to store the tree nodes in the appropriate order.
- Here, for example, is the routine for inorder traversal that uses a stack.

Non Recursive Traversal

```
void inorder(TreeNode<int>* root)
{
    Stack<TreeNode<int>* > stack;
    TreeNode<int>* p;
    p = root;
    do
    {
        while( p != NULL )
        {
            stack.push( p );
            p = p->getLeft();
        }
        // at this point, left tree is
  empty
```

Non Recursive Traversal

```
void inorder(TreeNode<int>* root)
{
    Stack<TreeNode<int>* > stack;
    TreeNode<int>* p;
    p = root;
    do
    {
        while( p != NULL )
        {
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            p = p->getLeft();
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        // at this point, left tree is
  empty
```

Non Recursive Traversal

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void inorder(TreeNode<int>* root)
{
    Stack<TreeNode<int>* > stack;
    TreeNode<int>* p;
    p = root;
    do
    {
        while( p != NULL )
        {
            stack.push( p );
            p = p->getLeft();
        }
        // at this point, left tree is
  empty
```

Non Recursive Traversal

```
        if( !stack.empty() )
        {
            p = stack.pop();
            cout << *(p->getInfo()) << " ";
            // go back & traverse right subtree
            p = p->getRight();
        }
    } while ( !stack.empty() || p != NULL );
}
```

Non Recursive Traversal

```
        if( !stack.empty() )
        {
            p = stack.pop();
            cout << *(p->getInfo()) << " ";
            // go back & traverse right subtree
            p = p->getRight();
        }
    } while ( !stack.empty() || p != NULL );
}
```

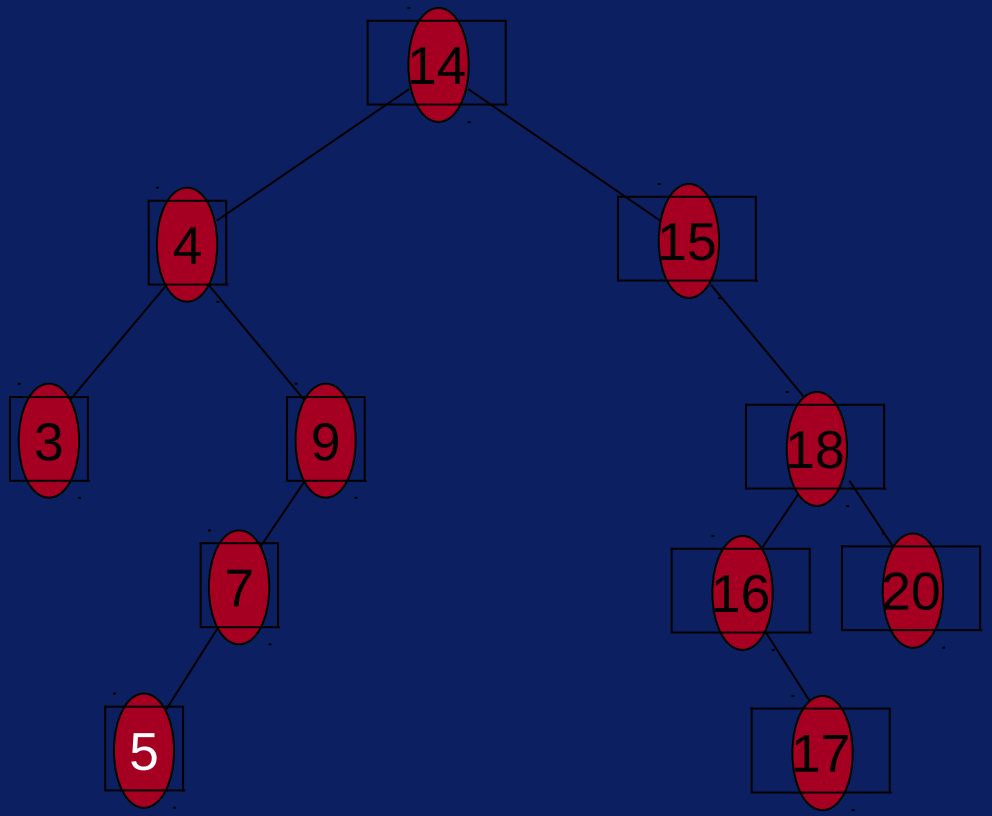
Non Recursive Traversal

```
        if( !stack.empty() )
        {
            p = stack.pop();
            cout << *(p->getInfo()) << " ";
            // go back & traverse right subtree
            p = p->getRight();
        }
    } while ( !stack.empty() || p != NULL );
}
```

Non Recursive Traversal

```
        if( !stack.empty() )
        {
            p = stack.pop();
            cout << *(p->getInfo()) << " ";
            // go back & traverse right subtree
            p = p->getRight();
        }
    } while ( !stack.empty() || p != NULL );
}
```

Nonrecursive Inorder




```
push(14)
..push(4)
....push(3)
3
4
..push(9)
....push(7)
......push(5)
5
7
9
14
push(15)
15
push(18)
..push(16)
16
..push(17)
17
18
push(20)
20
```

Traversal Trace

recursive inorder

```
inorder(14)
..inorder(4)
....inorder(3)
3
4
..inorder(9)
....inorder(7)
......inorder(5)
5
7
9
14
inorder(15)
15
inorder(18)
..inorder(16)
16
..inorder(17)
17
18
inorder(20)
20
```

nonrecursive inorder

```
push(14)
..push(4)
....push(3)
3
4
..push(9)
....push(7)
......push(5)
5
7
9
14
push(15)
15
push(18)
..push(16)
16
..push(17)
17
18
push(20)
20
```

Traversal Trace

recursive inorder

```
inorder(14)
..inorder(4)
....inorder(3)
3
4
..inorder(9)
....inorder(7)
......inorder(5)
5
7
9
14
inorder(15)
15
inorder(18)
..inorder(16)
16
..inorder(17)
17
18
inorder(20)
20
```

nonrecursive inorder

```
push(14)
..push(4)
....push(3)
3
4
..push(9)
....push(7)
......push(5)
5
7
9
14
push(15)
15
push(18)
..push(16)
16
..push(17)
17
18
push(20)
20
```

Traversal Trace

recursive inorder

```
inorder(14)
..inorder(4)
....inorder(3)
3
4
..inorder(9)
....inorder(7)
......inorder(5)
5
7
9
14
inorder(15)
15
inorder(18)
..inorder(16)
16
..inorder(17)
17
18
inorder(20)
20
```

nonrecursive inorder

```
push(14)
..push(4)
....push(3)
3
4
..push(9)
....push(7)
......push(5)
5
7
9
14
push(15)
15
push(18)
..push(16)
16
..push(17)
17
18
push(20)
20
```

Traversal Trace

recursive inorder

```
inorder(14)
..inorder(4)
....inorder(3)
3
4
..inorder(9)
....inorder(7)
......inorder(5)
5
7
9
14
inorder(15)
15
inorder(18)
..inorder(16)
16
..inorder(17)
17
18
inorder(20)
20
```

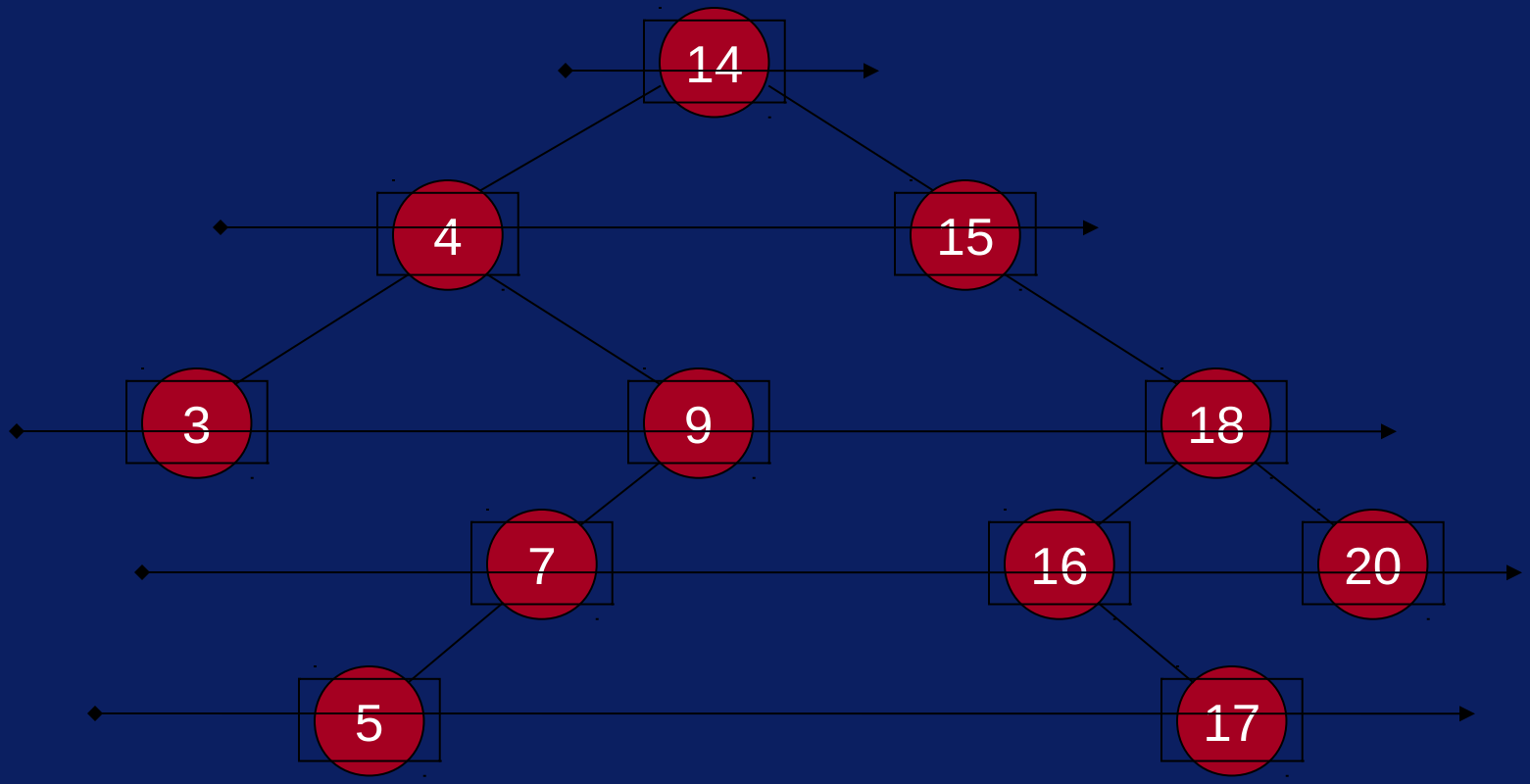
nonrecursive inorder

```
push(14)
..push(4)
....push(3)
3
4
..push(9)
....push(7)
......push(5)
5
7
9
14
push(15)
15
push(18)
..push(16)
16
..push(17)
17
18
push(20)
20
```

Level-order Traversal

- There is yet another way of traversing a binary tree that is not related to recursive traversal procedures discussed previously.
- In level-order traversal, we visit the nodes at each level before proceeding to the next level.
- At each level, we visit the nodes in a left-to-right order.

Level-order Traversal



Level-order: 14 4 15 3 9 18 7 16 20 5 17