CS-570 Computer Vision

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7. Corner Detection

Corners

- Just like edges, corners are perceptually important.
- More compact summary of an image since corners are fewer than edge pixels.
- A patch around a corner pixel is different from all other surrounding patches.

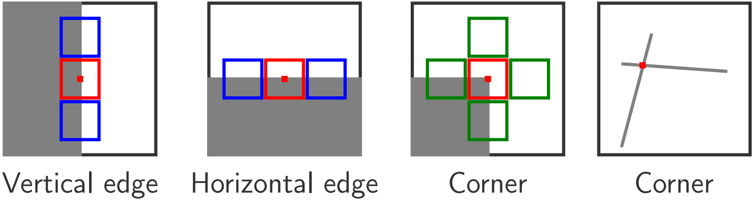


Figure: A patch containing a corner is different from all surrounding patches. Blue squares represent patches similar to the red patch. Green squares represent patches different from the red patch. Author: N. Khan (2021)

Corner Detection via Structure Tensor

- What do the eigenvalues of the structure tensor reveal about the local structure around a pixel?

$$\begin{aligned}
\lambda_{\text{large}} \approx \lambda_{\text{small}} \approx 0 &\implies \text{flat region} \\
\lambda_{\text{large}} \gg \lambda_{\text{small}} \approx 0 &\implies \text{edge} \\
\lambda_{\text{large}} > \lambda_{\text{small}} \gg 0 &\implies \text{corner}
\end{aligned}$$

- So a simple corner detection criterion could be $\lambda_{\text{small}} > \tau$.

Corner Detection via Structure Tensor

- But eigenvalue computation is a little expensive.
- Using the facts that
  1. $det(A) = A_{11}A_{22} - A_{12}^2 = \lambda_{\text{large}}\lambda_{\text{small}}$, and
  2. $trace(A) = A_{11} + A_{22} = \lambda_{\text{large}} + \lambda_{\text{small}}$

  popular corner detectors avoid eigenvalue computations.
- Popular corner detectors use a cornerness measure and then a detection criterion.

Method	Cornerness Measure	Detector
Harris	$\frac{det(A)}{trace(A)}$	$trace(A) > \tau$
Rohr	$det(A)$	$det(A) > \tau$

- To avoid multiple detections, non-maxima suppression must be performed on the cornerness values in 8-neighourhoods or larger.

Corner Detection
Algorithm

Input: Image I.
Parameters:
1) Noise smoothing scale σ,
2) Gradient smoothing scale ρ (should be greater than σ),
3) Threshold τ.

1. Compute Gaussian derivatives at noise smoothing scale σ
$$I_x = \frac{\partial G_\sigma}{\partial x} * I \quad \text{and} \quad I_y = \frac{\partial G_\sigma}{\partial y} * I$$
2. Compute the products
$$I_x^2, \quad I_y^2 \text{ and } \quad I_x I_y$$
3. Smooth the products at gradient smoothing scale ρ
$$G_\rho * I_x^2, \quad G_\rho * I_y^2 \quad \text{and} \quad G_\rho * I_x I_y$$
and construct structure tensor A at every pixel.

Corner Detection
Algorithm

4. Compute cornerness $C(i, j)$ at every pixel as

Harris	Rohr
$C_{ij} = \frac{A_{11}A_{22} - A_{12}^2}{A_{11} + A_{22}}$	$C_{ij} = A_{11}A_{22} - A_{12}^2$

5. Perform non-maxima suppression in 8-neighbourhood on cornerness image C.
6. Find corner pixels by thresholding remaining local maxima via

Harris	Rohr
$trace(A) = A_{11} + A_{22} > \tau$	$det(A) = A_{11}A_{22} - A_{12}^2 > \tau$

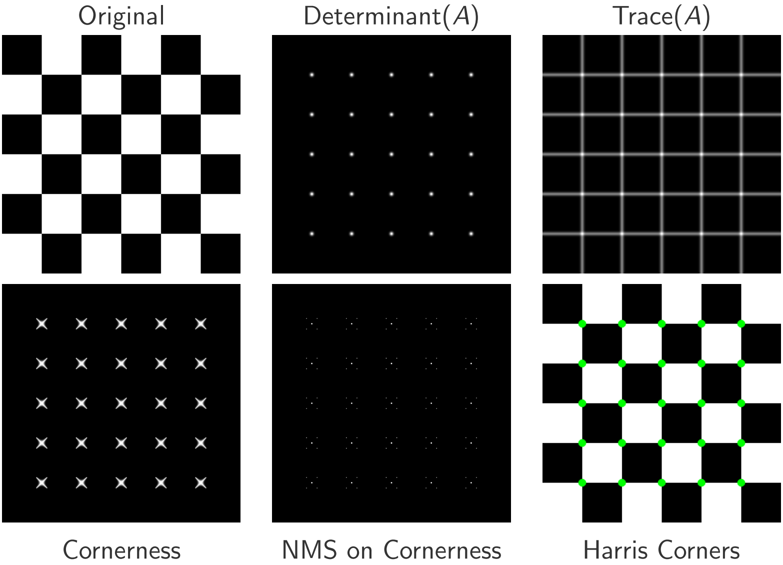


Figure: Harris corners detected with $\sigma = 0.2$, $\rho = 2$ and $\tau = $ 90th percentile of trace values. Author: N. Khan (2018)

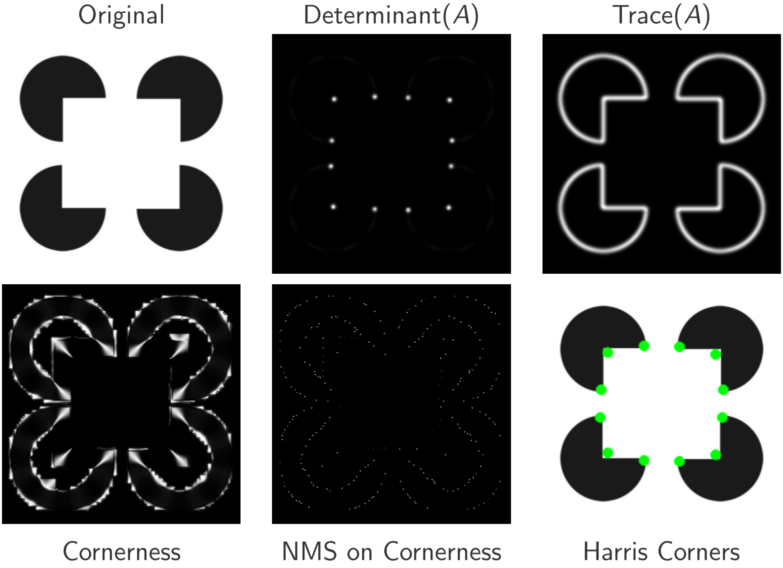


Figure: Harris corners detected with $\sigma = 0.5$, $\rho = 2$ and $\tau = 80$th percentile of trace values. Author: N. Khan (2018)

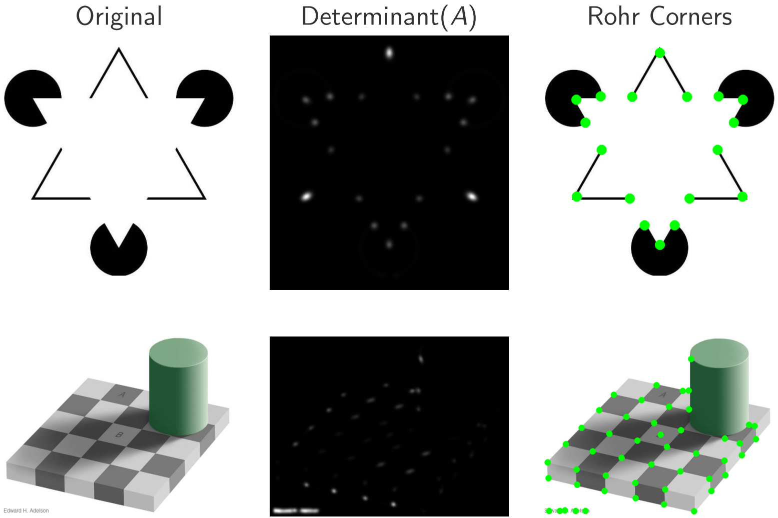


Figure: Corners detected by Rohr's method with $\sigma = 1$, $\rho = 6$ and $\tau = 98$th percentile of determinant values for **top row** and 95th for **bottom row**. Author: N. Khan (2018)

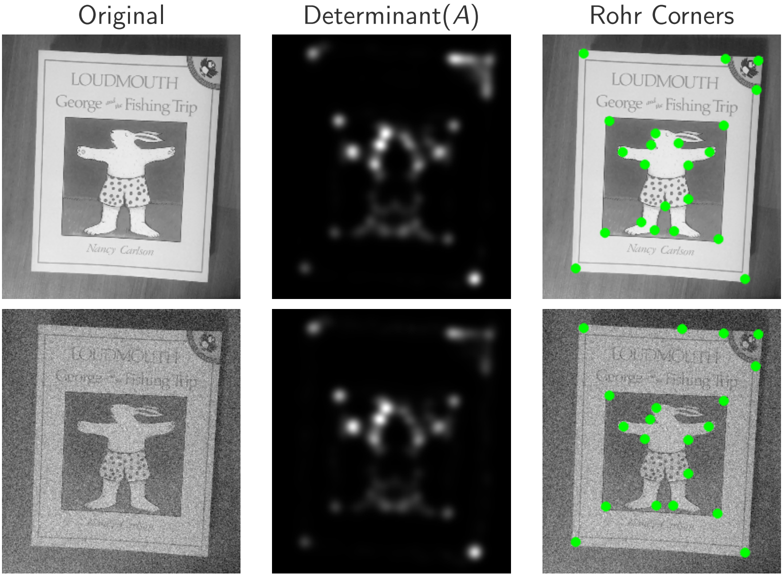
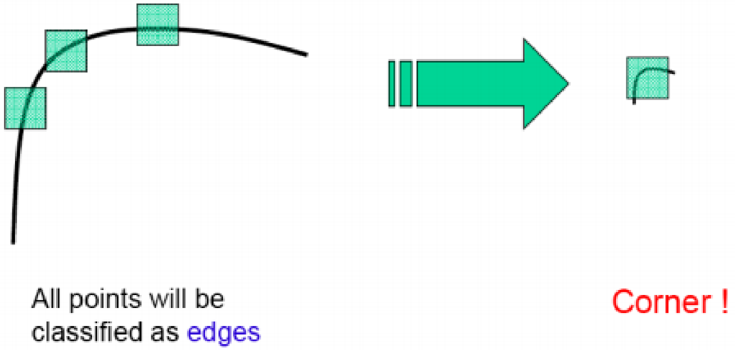


Figure: Corners detected by Rohr's method with $\rho = 6$ and $\tau =$ 95th percentile of determinant values. Noise smoothness scale was $\sigma = 3$ for **top row** and $\sigma = 4$ for **bottom row**. Author: N. Khan (2018)

Corners depend on scale



- Structure tensors and therefore corner detection are not scale invariant.
- Therefore, corner detection should take place at multiple scales.
- This leads to the concept of a *scale space*.

Scale Space via Gaussian Pyramids

Figure: A Gaussian pyramid with 3 levels and 5 smoothing scales. **Top to bottom**: Subsampling in both dimensions by factor 2^i for $i = 0, \ldots, 2$. **Left to right**: Gaussian blurring with $\sigma = \sqrt{2}^j \sigma_0$ for $j = 0, \ldots, 4$ and $\sigma_0 = \sqrt{2}$. Author: N. Khan (2018)

Scale Space via Gaussian Pyramids

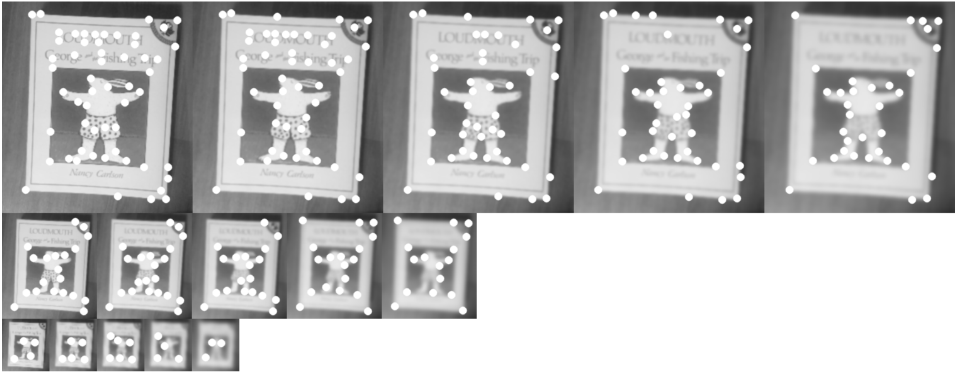


Figure: Corner detection in scale space obtained via Gaussian pyramids. Some corners are detected only at certain resolutions and certain smoothness scales. Corners that *persist across resolutions and smoothness scales* are called strong or stable corners. Author: N. Khan (2018)

Scale Space via Gaussian Pyramids

function makeGaussianPyramid(I,num_levels,num_scales,k,σ_0)
for $i = 0$ to num_levels-1
    $J =$ subsample($I, \frac{1}{2^i}$)
    for $s = 0$ to num_scales-1
        $\sigma = k^s \sigma_0$
        $GP[i, s] = J * G_\sigma$