Name: ____________________ Roll Number: ____________________

1. (1 point) Prove that using a 1st order Taylor's expansion for $f(x+h)$ yields

$$f'(x) = \frac{f(x+h) - f(x)}{h} + O(h)$$

This is a derivative approximation using forward difference.

2. (1 point) Prove that using a 1st order Taylor's expansion for $f(x-h)$ yields

$$f'(x) = \frac{f(x) - f(x-h)}{h} + O(h)$$

This is a derivative approximation using backward difference.

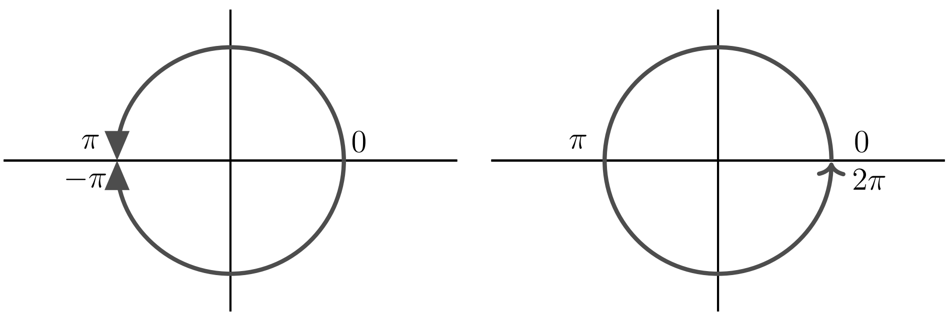
3. For edge detection in an image I,

   (a) Using the partial derivative values I_x and I_y, write down the formulae for computing

      i. (1 point) gradient magnitude

      ii. (1 point) gradient direction

   (b) The atan2 function returns angle in the range $\theta \in (-\pi, \pi)$ as shown in Figure 1(a).



(a) (b)

Figure 1: Ranges for gradient direction.

      i. (4 points) To quantize angle $\theta \in (-\pi, \pi)$ into 8 uniformly spaced bins, fill in the blanks for appropriate ranges of θ. **Hint**: The range $(-\pi, \pi)$ has been divided into 8 bins of the same size. Consider the range of angles provided below for the bins $q(\theta) = 0$ and $q(\theta) = 4$ and use them to compute the angle ranges for other bins.

$$q(\theta) = \begin{cases} 0 & \text{if } -\frac{\pi}{8} < \theta \leq \frac{\pi}{8} \\ 1 & \text{if } \underline{\hspace{2cm}} < \theta \leq \underline{\hspace{2cm}} \\ 2 & \text{if } \underline{\hspace{2cm}} < \theta \leq \underline{\hspace{2cm}} \\ 3 & \text{if } \underline{\hspace{2cm}} < \theta \leq \underline{\hspace{2cm}} \\ 4 & \text{if } |\theta| \geq \frac{7\pi}{8} \\ 5 & \text{if } \underline{\hspace{2cm}} < \theta \leq \underline{\hspace{2cm}} \\ 6 & \text{if } \underline{\hspace{2cm}} < \theta \leq \underline{\hspace{2cm}} \\ 7 & \text{if } \underline{\hspace{2cm}} < \theta \leq -\frac{\pi}{8} \end{cases}$$

      ii. (2 points) How can we convert angles returned by atan2 in the range $\theta \in (-\pi, \pi)$ to the range $\theta \in (0, 2\pi)$ measured in anti-clockwise direction as shown in Figure 1(b)?