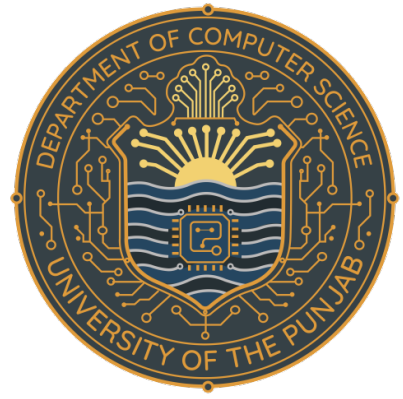
CS-866 Deep Reinforcement Learning

Improved Policy-Based Learning I: Actor-Critic Methods



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Improving Policy Gradients: Reducing High Variance

- Vanilla (episodic) policy gradient methods suffer from **high variance**.
- **Key Enhancements to Reduce Variance:**
  - **Actor-Critic**: Add a value-based critic using TD bootstrapping to guide updates
  - **Baseline Subtraction**: Use an *advantage function* to reduce variance of returns
  - **Trust Regions**: Constrain large policy updates for stability (e.g., TRPO, PPO)
  - **Exploration Strategies**: Encourage high-entropy policies to escape local minima
- These techniques dramatically improved the practicality and performance of policy-based RL.

Actor-Critic Bootstrapping: Motivation

Actor-Critic combines policy-based and value-based methods:

- **Actor**: policy $\pi_\theta(a|s)$ (chooses actions)
- **Critic**: value function $V_\phi(s)$ (evaluates states/actions)

Why Actor-Critic?

- REINFORCE: low bias but very high variance
- Full-episode sampling $\Rightarrow$ updates vary wildly
- Actor-Critic keeps low bias, but reduces variance via value bootstrapping

Actor-Critic has become a core RL paradigm (A3C, PPO, SAC, DDPG, ...).

Where Does Variance Come From?

Sources of variance in policy gradients

1. **Cumulative return variance**: Full episode rewards vary greatly
2. **Gradient estimate variance**: Stochastic action samples $\Rightarrow$ noisy gradient

Solutions:

- **Bootstrapping** for lower reward variance
- **Baseline subtraction** (advantage) for lower gradient variance

The critic $V_\phi(s)$ supplies both bootstrapping and baselines.

Network Structure: Actor and Critic

Value function used in Actor-Critic:

$$V_\phi(s)$$

Parameterization options

- Separate networks: θ (actor), ϕ (critic)
- Shared body + two heads (policy head + value head)

Notation:

- Policy parameters θ
- Value-network parameters ϕ

Actor improves policy; critic provides value estimates for stability.

Temporal Difference Bootstrapping

Sampling full episodes gives high variance: many possible trajectories $\Rightarrow$ updates unstable.

Bootstrapping idea:

- Use TD targets to estimate returns before episode ends
- n-step bootstrapped target

$$\hat{Q}_n(s_t, a_t) = \sum_{k=0}^{n-1} \gamma^k r_{t+k} + \gamma^n V_\phi(s_{t+n})$$

- Interpolates between MC (high variance) and TD (higher bias)

Actor-Critic: Learning Updates

Value function loss:

$$\mathcal{L}(\phi) = \left(\hat{Q}_n(s_t, a_t) - V_\phi(s_t)\right)^2$$

Policy gradient update:

$$\nabla_\theta J(\theta) = \hat{Q}_n(s_t, a_t) \nabla_\theta \log \pi_\theta(a_t|s_t)$$

Policy uses bootstrapped estimate instead of full return R.

Actor-Critic with Bootstrapping: Pseudocode

Algorithm 1 Actor-Critic with n-step Bootstrapping

Initialize policy π_θ and value network V_ϕ
repeat
  for each episode $i = 1..M$ **do**
    Sample trajectory $\tau = \{s_0, a_0, r_0, \ldots, s_T\}$
    for $t = 0..T-1$ **do**
      $\hat{Q}_n(s_t, a_t) = \sum_{k=0}^{n-1} \gamma^k r_{t+k} + \gamma^n V_\phi(s_{t+n})$ ▷ MC + n-step TD
    end for
  end for
  $\phi \leftarrow \phi - \alpha \nabla_\phi \sum_t (\hat{Q}_n - V_\phi(s_t))^2$ ▷ Updates in batch mode
  $\theta \leftarrow \theta + \alpha \sum_t [\hat{Q}_n \nabla_\theta \log \pi_\theta(a_t|s_t)]$
until convergence

Key Takeaways

Actor-Critic Summary

- Actor learns policy; critic learns value function
- Reduces variance vs. vanilla policy gradients
- Bootstrapping $\Rightarrow$ better value estimates
- Uses n-step returns between MC and TD
- Foundation for modern RL (A3C, PPO, SAC, etc.)

Baseline Subtraction with Advantage Function

Goal: Reduce variance of policy gradient estimates.

Key idea: Subtract a baseline from returns to lower variance *without changing the expectation*.

Example:

- Action returns in a state: 65, 70, 75
- All positive $\rightarrow$ vanilla PG pushes **all** action probabilities up
- Better: push above-average actions up (75) and below-average down (65)

Baseline: Use value function $V(s)$

$$A(s, a) = Q(s, a) - V(s)$$

Advantage: Measures improvement of a over expected value of s.

Advantage Function in Practice

Combine bootstrap estimate with baseline:

$$\hat{A}_n(s_t, a_t) = \hat{Q}_n(s_t, a_t) - V_\phi(s_t)$$

$$\nabla_\theta J(\theta) = \hat{A}_n(s_t, a_t) \nabla_\theta \log \pi_\theta(a_t|s_t)$$

Interpretation:

- Positive advantage $\rightarrow$ increase probability of action
- Negative advantage $\rightarrow$ decrease probability of action
- Zero advantage $\rightarrow$ no change

Actor-Critic with Baseline + Bootstrapping

Actor critic: Policy + value function

Algorithm structure:

- Collect trajectory
- Compute n-step target and advantage
- Update critic (value function)
- Update actor (policy)

Losses:

$$\mathcal{L}_V = (\hat{A}_n)^2 \qquad \mathcal{L}_\pi = -\hat{A}_n \log \pi_\theta(a|s)$$

Pseudocode: Actor-Critic with Advantage

Initialize policy π_θ, value function V_ϕ
while not converged **do**
    for each episode **do**
        Collect trajectory τ
        for each time t **do**
            $\hat{Q}_n = \sum_{k=0}^{n-1} \gamma^k r_{t+k} + \gamma^n V_\phi(s_{t+n})$
            $\hat{A}_n = \hat{Q}_n - V_\phi(s_t)$
        end for
    end for
    $\phi \leftarrow \phi - \alpha \nabla_\phi \sum_t \hat{A}_n^2$
    $\theta \leftarrow \theta + \alpha \sum_t \hat{A}_n \nabla_\theta \log \pi_\theta(a_t|s_t)$
end while

General Policy Gradient Formulation

$$\nabla_\theta J(\theta) = \mathbb{E}\left[\sum_t \Psi_t \nabla_\theta \log \pi_\theta(a_t|s_t)\right]$$

Choices for target Ψ_t:

$\Psi_t = \hat{Q}_{MC}$	Monte Carlo
$\Psi_t = \hat{Q}_n$	n-step bootstrapping
$\Psi_t = \hat{A}_{MC}$	Advantage (MC)
$\Psi_t = \hat{A}_n$	Advantage + bootstrapping
$\Psi_t = Q_\phi(s, a)$	Critic estimated Q

A3C (Asynchronous Advantage Actor-Critic)

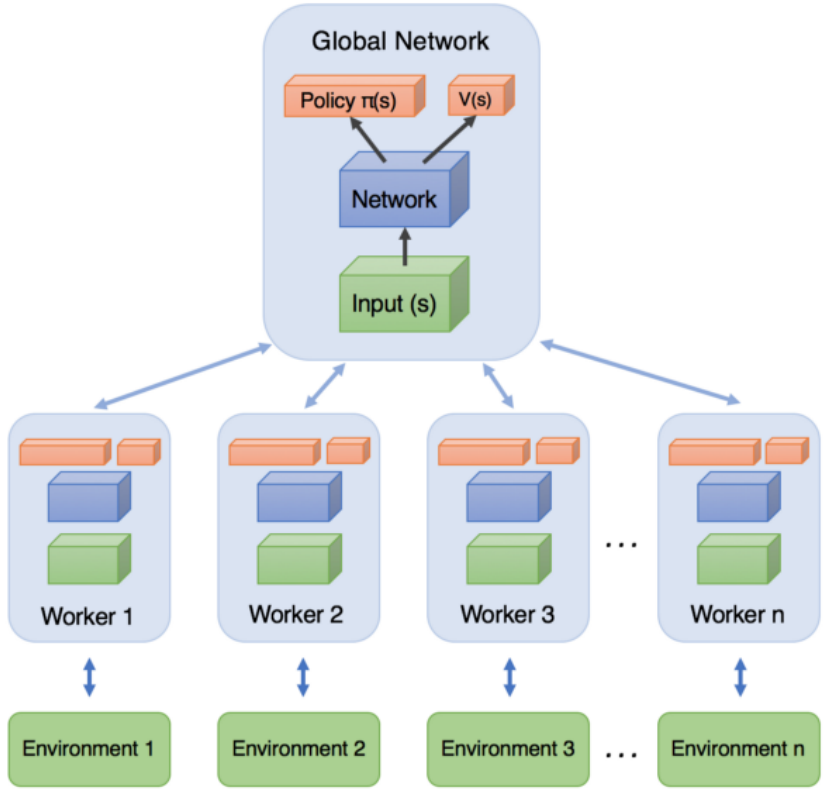
Why A3C?

- Extends advantage actor-critic
- Uses many parallel agents to stabilize learning
- Neural networks estimate both V and A and π
- Asynchronous updates to shared parameters

Benefits:

- Efficient experience collection
- Reduced correlation between samples
- Strong performance on Atari and continuous control

A3C Architecture



- Shared CNN feature extractor
- Separate value and policy heads
- Parallel actors update global params asynchronously