Name: ______________________ Roll Number: ______________________

1. For vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^d$ and matrices $\mathbf{M} \in \mathbb{R}^{k \times d}$ and $\mathbf{A} \in \mathbb{R}^{d \times d}$, prove the following derivatives.
   (a) (2 points) $\nabla_{\mathbf{x}}(\mathbf{y}^T\mathbf{x}) = \nabla_{\mathbf{x}}(\mathbf{x}^T\mathbf{y}) = \mathbf{y}$
   Hint: You may want to write out the complete expression for the dot-product $\mathbf{x}^T\mathbf{y}$. This expression will be a scalar value. You will need to take derivatives of this scalar value with respect to every element of $\mathbf{x}$.
   (b) (3 points) $\nabla_{\mathbf{x}}(\mathbf{M}\mathbf{x}) = \mathbf{M}^T$
   Hint: You may denote the i-th row of matrix $\mathbf{M}$ by $\mathbf{m}_i^T$. Then use part (a) to write the derivative of the expression $\mathbf{m}_i^T\mathbf{x}$.
   (c) (3 points) $\nabla_{\mathbf{x}}(\mathbf{x}^T\mathbf{A}\mathbf{x}) = (\mathbf{A} + \mathbf{A}^T)\mathbf{x}$
   Hint: You may use the product rule of differentiation. When applied to vectors, the rule states that

$$\nabla_{\mathbf{x}}\left(\mathbf{u}^T\mathbf{v}\right) = \nabla_{\mathbf{x}}\left(\mathbf{u}\right)\mathbf{v} + \nabla_{\mathbf{x}}\left(\mathbf{v}\right)\mathbf{u}$$

   where both $\mathbf{u}$ and $\mathbf{v}$ are functions of $\mathbf{x}$. You may also use the derivative from part (b).
   (d) (2 points) $\nabla_{\mathbf{x}}(\mathbf{x}^T\mathbf{A}\mathbf{x}) = 2\mathbf{A}\mathbf{x}$ when $\mathbf{A}$ is symmetric