Name: ______________________ Roll Number: ______________________

1. What is the goal of Maximum Likelihood Estimation (MLE) in Gaussian density estimation?
   A. To find the parameter values that minimize the variance of the data.
   B. To find the parameter values that maximize the likelihood of observing the given data.
   C. To minimize the mean squared error of the model.
   D. To find the parameter values that minimize the loss function.

   Answer: B) To find the parameter values that maximize the likelihood of observing the given data.

2. How do you typically solve for the MLE of the mean μ and variance σ^2 for a Gaussian distribution?
   A. By maximizing the posterior distribution.
   B. By differentiating the log-likelihood function with respect to μ and σ^2 and setting the derivatives to zero.
   C. By minimizing the squared error between the true and predicted values.
   D. By using gradient descent to find the values of μ and σ^2.

   Answer: B) By differentiating the log-likelihood function with respect to μ and σ^2 and setting the derivatives to zero.

3. Which of the following is **true** about the MLE estimates for the mean and variance of a Gaussian distribution?
   A. The MLE estimate for the mean is the median of the data.
   B. The MLE estimate for the variance cannot be found.
   C. The MLE estimate for the mean is the sample mean of the data.
   D. The MLE estimate for the variance is the sample standard deviation.

   Answer: C) The MLE estimate for the mean is the sample mean of the data.

4. Given a dataset $X = \{x_1, x_2, \ldots, x_N\}$ and the assumption that the data comes independently from a Gaussian distribution with mean μ and variance σ^2, the likelihood function is written as

$$L(\mu, \sigma^2) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x_i - \mu)^2}{2\sigma^2}\right)$$

   What is the log-likelihood function?
   A. $\log L(\mu, \sigma^2) = -N \log(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum_{i=1}^{N} (x_i - \mu)^2$
   B. $\log L(\mu, \sigma^2) = -\frac{N}{2} \log(2\pi) - \frac{N}{2} \log(\sigma^2) - \frac{1}{2\sigma^2} \sum_{i=1}^{N} (x_i - \mu)^2$
   C. $\log L(\mu, \sigma^2) = -\frac{1}{2} \sum_{i=1}^{N} (x_i - \mu)^2$
   D. $\log L(\mu, \sigma^2) = \frac{1}{2} \sum_{i=1}^{N} \log(2\pi\sigma^2) - (x_i - \mu)$

   Answer: B) $\log L(\mu, \sigma^2) = -\frac{N}{2} \log(2\pi) - \frac{N}{2} \log(\sigma^2) - \frac{1}{2\sigma^2} \sum_{i=1}^{N} (x_i - \mu)^2$

5. In Gaussian density estimation, what does the log-likelihood function allow us to do that the likelihood function does not?
   A. Find the mean of the data

B. The log-likelihood transforms products into sums, making it easier to differentiate and avoids products reaching zero.

C. Make the computation faster by eliminating the variance

D. Increase the likelihood of the data

Answer: B) The log-likelihood transforms products into sums, making it easier to differentiate. It also avoids products of probabilities reaching zero.