SE 461 Computer Vision – Assignment 2

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November 24, 2014
Due Date: Monday, 1st December, 2014 before class.

Fourier Transform Theoretical

Please refer to Appendix A on page 4 for help on this part.

1. For a complex number $z = a + bi$, compute $z * z$ and $z * \bar{z}$. Which one yields the squared norm $|z|^2 = a^2 + b^2$?

2. Verify the relationship $\theta = \omega t$ between angular distance θ, angular speed ω and time t.

3. Verify the relationship $\omega = 2\pi f$ between angular speed ω and angular frequency f.

4. For a complex vector $\mathbf{f} = (f_1, \ldots, f_N)$, compute $\mathbf{f}^T\mathbf{f}$ and $\mathbf{f}^T\bar{\mathbf{f}}$. Which one yields the squared norm of of vector $\mathbf{f}$ (given by $|\mathbf{f}|^2 = |f_1|^2 + \cdots + |f_N|^2$)?

5. Prove orthonormality of the Fourier basis. That is, given any two basis vectors $\mathbf{b}_i, \mathbf{b}_j$, prove that

$$\mathbf{b}_i^T \bar{\mathbf{b}}_j = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \tag{1}$$

   Considering that the Fourier basis is orthonormal, do the different frequencies interfere with each other in representing the signal?

6. Let $\mathbf{f} = (6, 5, 4, 1)^T$ be a signal with 4 values.

   (a) Compute the 4 discrete Fourier basis vectors $\mathbf{b}_0, \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$.

   (b) Compute the coefficients of $\mathbf{f}$ in the discrete Fourier basis.

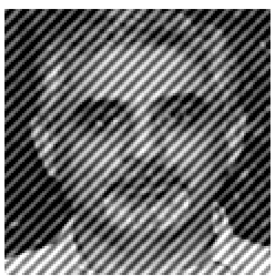
   (c) Compute the reconstruction of $\mathbf{f}$ from these coefficients. Is it equal to the original signal $\mathbf{f}$?

Fourier Transform Programming

1. In the function **myAffineRescaling.m** add the code that performs an Affine Grayscale Transformation of the input image between 0 and c.

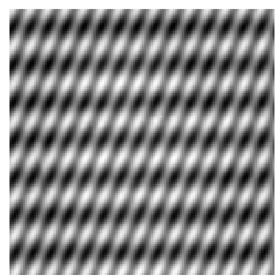
2. In the function **myLogDynamicCompression.m** add the code that performs Logarithmic Dynamic Compression of the input image between 0 and c.

3. My boss has been abducted and jailed in a funny looking jail as shown in **bossJailed1.png**. I have heard that a strange space exists where getting my boss out of jail is very easy. I have been able to obtain some MATLAB code for going to that strange space and then coming back. This code is present in **unjail.m**. Can you help me free my boss in **bossJailed1.png** by removing the jail bars? Store the result in **bossFreed1.png**.



bossJailed1.png

bossJailed2.png



mystery.png

4. The abducters might have placed my boss in a maximum security prison as shown in **bossJailed2.png**. Can you free him from there too? Store the result in **bossFreed2.png**.

5. Impressed by your abilities to free captives, the abductors have challenged you to free a mystery captive in **mystery.png**. Can you free this captive as well? Who is the captive? Store the result in **Captive'sFirstNameFreed.png**.

Submission

This assignment is to be done in groups of 3 students each. **It is highly recommended that you try this assignment individually at first and then combine your results**. Email your assignment to the TA Nausheen

Qaiser at **phdcsf13m005@pucit.edu.pk** as a .zip file with the naming convention

`RollNumber1_RollNumber2_RollNumber3_YourName_Assignment2.zip`

For example, if roll numbers of your group members are BSEF11M997, BSEF11M998 and BSEF11M999, then the .zip file should be named

`BSEF11M997_BSEF11M998_BSEF11M999_Assignment2.zip`

The .zip file should contain the following directories:

- **Theoretical**
- **Programming**

The **Theoretical** directory should contain the following:

1. A .txt/doc/pdf file called README.txt/doc/pdf containing your answers. If you want to write the answers by hand, then a digital photograph or scanned copy of your answers should be placed here.

The **Programming** directory should contain the following:

1. The files **myAffineRescaling.m** and **myLogDynamicCompression.m** supplemented with the missing code.
2. The file **unjail.m** supplemented with the missing code.
3. The images
   (a) **bossFreed1.png**,
   (b) **bossFreed2.png**, and
   (c) **Captive'sFirstNameFreed.png**.
4. A .txt file called README.txt describing how you managed to free all the captives. This should also include the identity of the mystery captive.

Note: To submit your results in a single, beautiful looking .pdf file, the LaTeX source for this document is also provided in the Assignment2.tex file. You can use the command \answer{} to fill in your answers below each question. Please consult your instructor or TA for more help. **Remember: Word is ugly and LaTeX is beautiful!**

A 1D Discrete Fourier Transform

The 1D discrete Fourier transform (DFT) of a **finite**, **sampled** signal $f = (f_0, \ldots, f_{M-1})^T$ with finite extent is given by

$$\hat{f}_u = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} f_m e^{-i2\pi u \frac{m}{M}} \tag{2}$$

for frequencies $u = (0, \ldots, M-1)$. A signal with M values is decomposed into M frequency coefficients. The corresponding 1D inverse discrete Fourier transform is given by

$$f_m = \frac{1}{\sqrt{M}} \sum_{u=0}^{M-1} \hat{f}_u e^{i2\pi u \frac{m}{M}} \tag{3}$$

for $m = (0, \ldots, M-1)$.

A.1 Interpretation as change of basis

The Fourier basis is given by

$$\mathbf{b}_u = \frac{1}{\sqrt{M}} (e^{-i2\pi u \frac{0}{M}}, e^{-i2\pi u \frac{1}{M}}, \ldots, e^{-i2\pi u \frac{M-1}{M}})^T \tag{4}$$

for frequencies $u = (0, \ldots, M-1)$. A signal $\mathbf{f}$ can be projected onto a basis vector $\mathbf{b}$ via the inner-product

$$< \mathbf{f}, \mathbf{b} >= \sum_{m=0}^{M-1} f_m \bar{b}_m \tag{5}$$

The DFT computes the Fourier basis coefficients $\hat{f}_u =< \mathbf{f}, \mathbf{b}_u >$ for frequencies $u = (0, \ldots, M-1)$. The inverse DFT reconstructs the signal from the Fourier basis coefficients via $\mathbf{f} = \sum_{u=0}^{M-1} \hat{f}_u \mathbf{b}_u$.

A.2 Proving orthogonality of Fourier basis vectors

You might need the formula for the sum of a geometric series

$$\sum_{m=0}^{M-1} r^m = \frac{1 - r^M}{1 - r} \tag{6}$$