SE 461 Computer Vision

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PUCIT

Lecture 13 and 14

Note

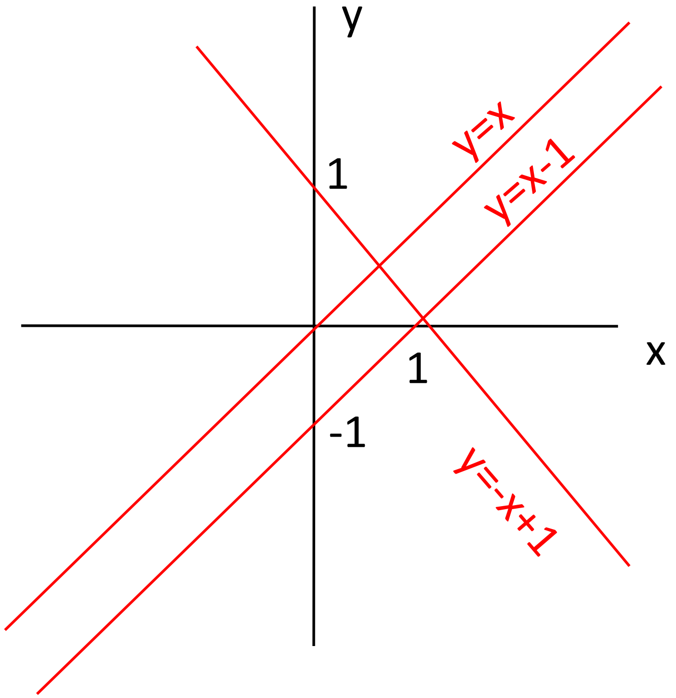
1. Missing assignments/quizes are unacceptable.
   - Please contact me if you have not submitted any assignment or quiz.
2. Follow submission instructions carefully.
3. You will learn **only by implementing**.
   - Explore/verify/reject the ideas covered in class by writing small Matlab codes.
   - The lectures cover the basic ideas – implementation details are sometimes as important as the idea.
   - Some students are doing this. So don’t rationalise your laziness!

Hough Transform for Line Detection

- A powerful method for detecting curves from boundary information.
- Exploits the duality between points on a curve and parameters of the curve.
- Can detect analytic as well as non-analytic curves.

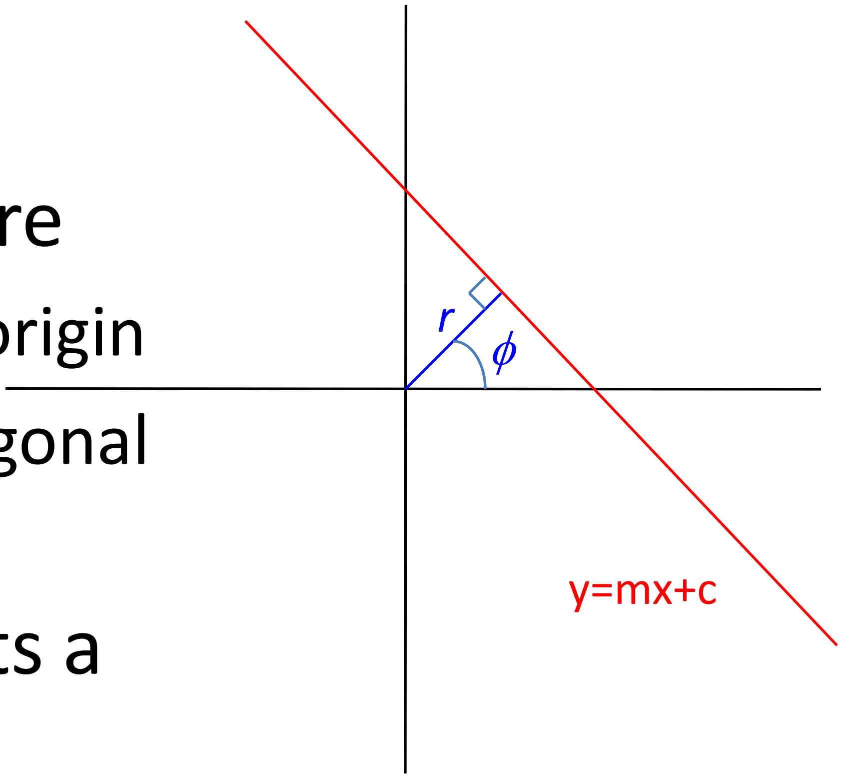
Analytic Representation of a Line

- Analytic Representation
  - Line: y=mx+c
- Every choice of parameters (*m*,*c*) represents a different line.
- This is known as the <u>slope-intercept parameter space</u>.
- Weakness: vertical lines have m=infinity.



Polar Representation

- Solution: Polar representation (r, ϕ) where
  - r = distance of line from origin
  - ϕ = angle of vector orthogonal to the line
- Every (r, ϕ) pair represents a 2D line.



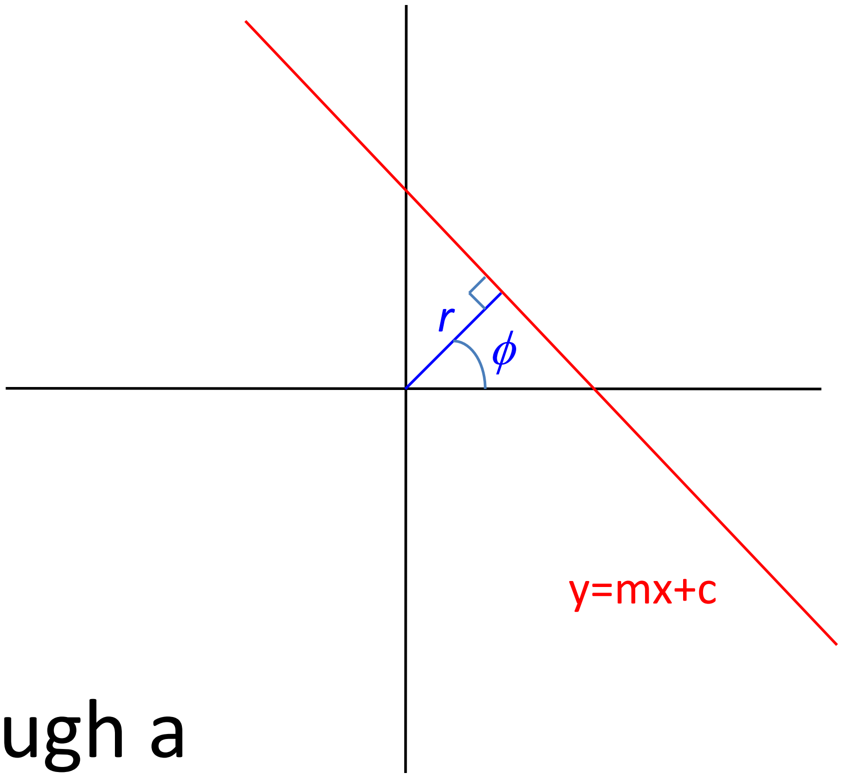
Polar Representation

- Cartesian to Polar

$$y = mx + c$$

$$y = -\frac{\cos(\theta)}{\sin(\theta)}x + \frac{r}{\sin(\theta)}$$

$$r = x\cos(\theta) + y\sin(\theta)$$



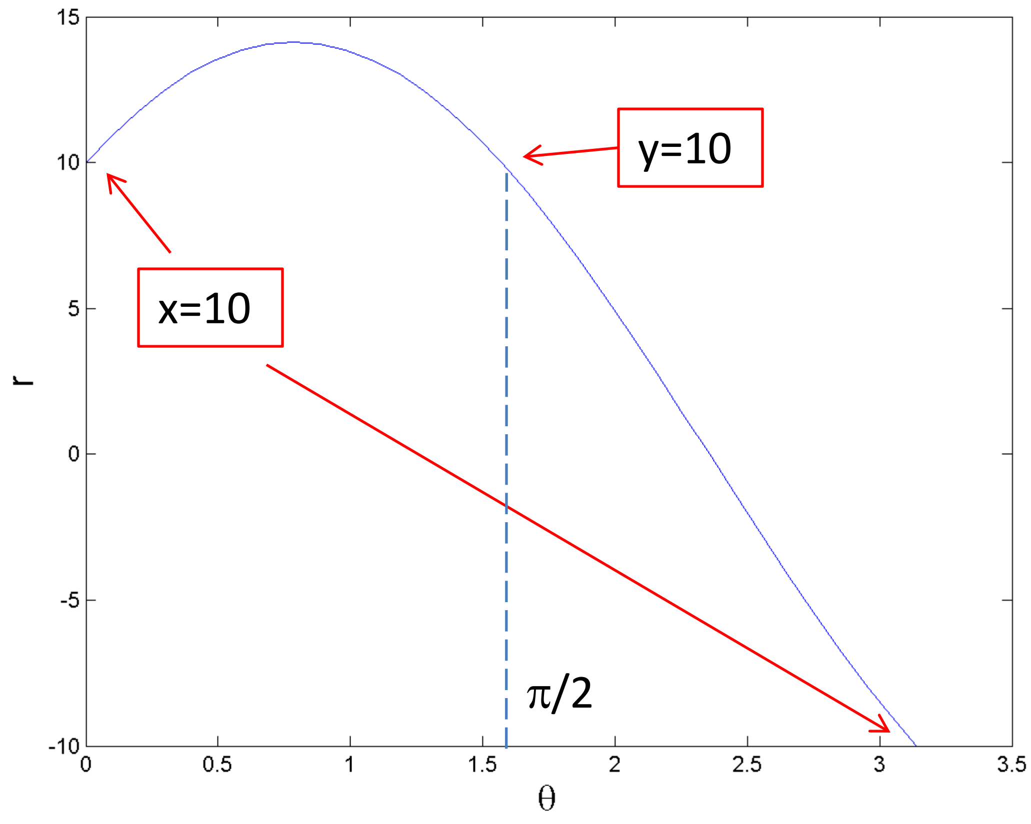
- **Key insight**: If a line through a known point (*x*,*y*) has angle φ, how can we find *r*?

Generating all possible lines through a point (x,y)

```
x=10;
y=10;
theta=0:pi/32:pi;
r=x*cos(theta)+y*sin(theta);
plot(theta,r);
```

In the space (r,φ) of polar parameters, the light blue curve represents **all lines** that can pass through the point (10,10).

We can generate lines through (x,y) by varying φ and computing the corresponding r-value.



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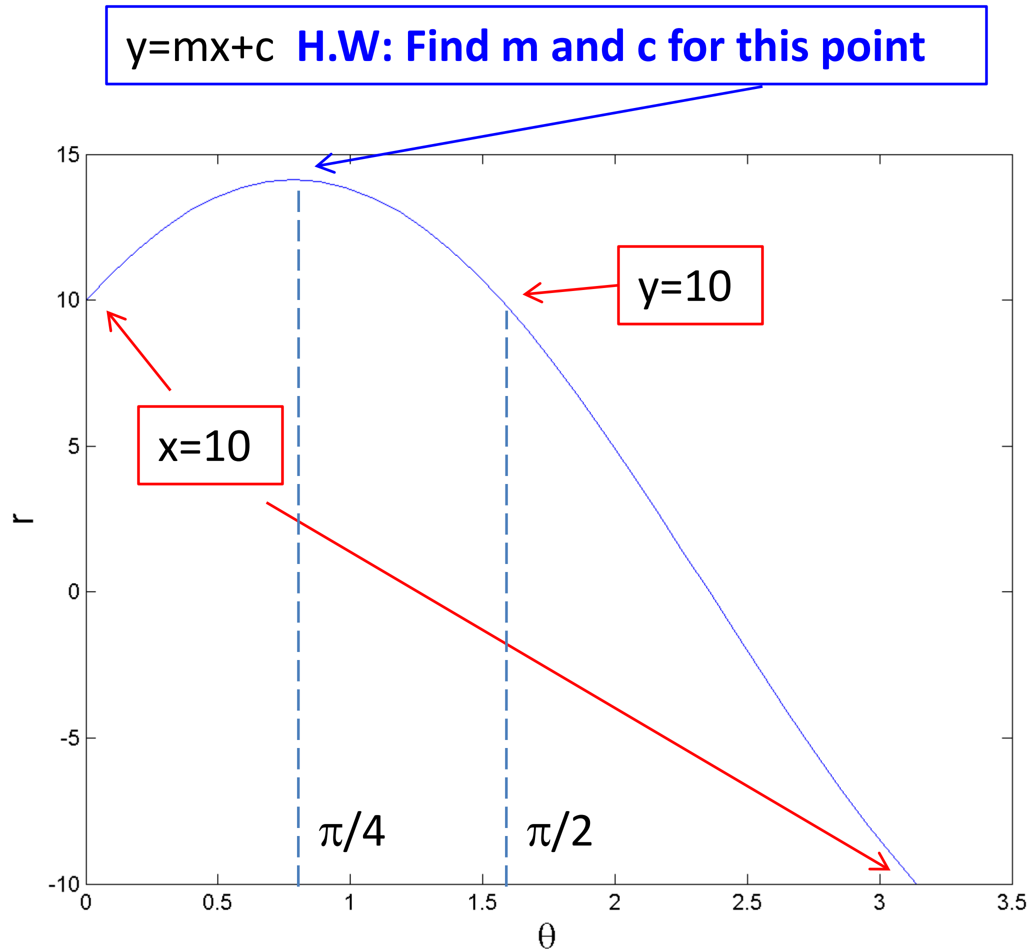

Hough Transform for Line Detection

- All lines going through a point (x,y) can be generated by iterating over $\phi=[0,\pi]$ and computing the corresponding $r(\phi)$.
  - That is, all lines going through a point (x,y) satisfy $r(\phi) = x.\cos(\phi)+y.\sin(\phi)$.
- So given any edge point (x,y), iterate over $\phi=[0,\pi]$ and generate the pair $(r(\phi),\phi)$.
  - The point (x,y) votes for all lines $(r(\phi),\phi)$ that pass through it.
- Valid lines can be detected by thresholding the votes .

Hough Transform for Line Detection

Initialise (vote) **accumulator array** *A* to all zeros.

For every edge point (x,y)

    For $\phi = 0$ to π

        Compute $r = x.\cos(\phi) + y.\sin(\phi)$

        Increment $A(r,\phi)$ by 1 <--- vote of point (x,y) for line (r,ϕ)

    EndFor

EndFor

Valid lines are where $A >$ threshold

Hough Transform

- **Improvement 1**: After edge detection, we already know the gradient direction at (*x,y*).
  - So there is no need to iterate over all possible ϕ =[*0,*π]. Use the correct ϕ from the gradient direction.
- **Improvement 2**: Smooth the accumulator array *A* to account for uncertainties in the gradient direction.

Hough Transform for Circle Detection

- Analytic representation of circle of radius r centered at (a,b) is $(x-a)^2+(y-b)^2-r^2=0$
- Hough space has 3 parameters (a,b,r)

For every boundary point (x,y)

For every (a,b) in image plane

Compute r(a,b)

Increment A(a,b,r) by 1

A>threshold represents valid circles.

What if we know the gradient direction at (x,y)?

Hough Transform for Circle Detection

- If we know the gradient direction g(x,y) at point (x,y), then we also know that the center (a,b) can only lie along this line
- Hough space still has 3 parameters (a,b,r) but we search for r over a 1D space instead of a 2D plane.

For every boundary point (x,y)

    For every (a,b) along gradient direction g(x,y)

        Compute r

        Increment A(a,b,r) by 1

A>threshold represents valid circles.

Hough Transform

- Any analytic curve (represented in the form f(x)=0) can be detected using the Hough transform.
  - LINE: $r = x\cos\theta + y\sin\theta$
  - CIRCLE: $x_0 = x - r\cos\theta$ where θ is gradient direction
    $y_0 = y - r\sin\theta$
  - ELLIPSE: $x_0 = x - a\cos\theta$ where θ is gradient direction
    $y_0 = y - b\sin\theta$
  - GENERAL: $f(\mathbf{x}, \mathbf{params}) = 0$

Hough Transform

- Hough space $param_1 \times param_2 \times \ldots \times param_N$ becomes very large when number of parameters N is increased.
- Using orientation information g(x,y) in addition to positional information (x,y) leads to a smaller search space.

Generalized Hough Transform

- When shape is non-analytic.
  - Can’t be represented as f(**x,params**)=0

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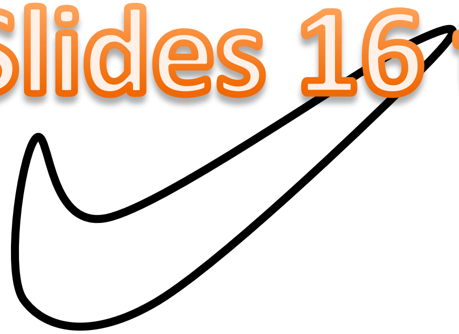

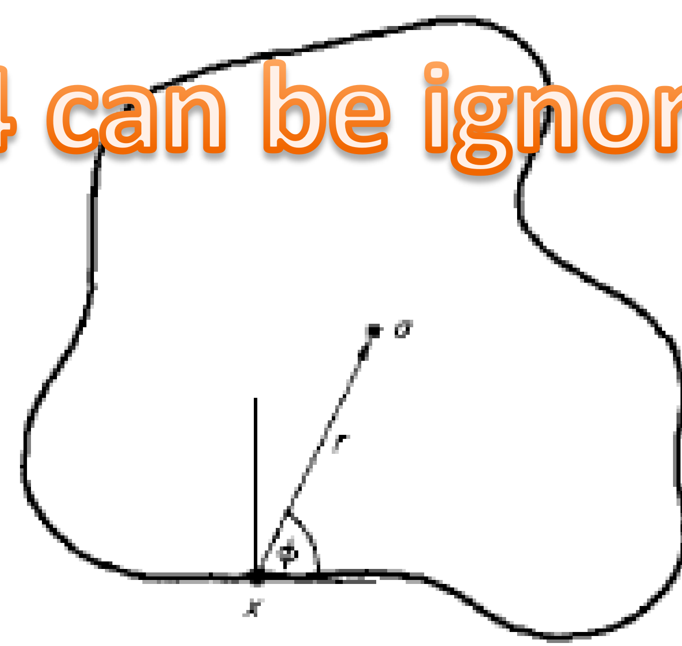

Fig. 6. Geometry for generalized Hough transform.

Generalized Hough Transform

- Training
  - A representation of shape of interest is built in the form of an R-Table
- Detection

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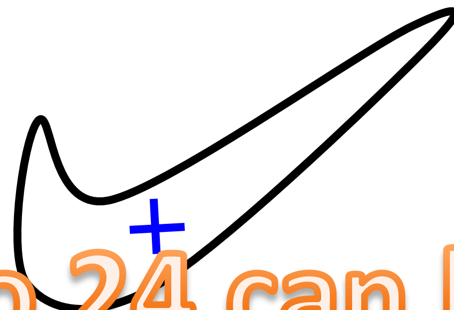
  - Using R-Table, a given shape is matched to the shape of interest

GHT - Training

- Given the shape of interest



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- Find Centroid (x_c, y_c) of shape
  - Centroid (x_c, y_c) = average of all boundary points

GHT - Training

- Find r = (x', y') for each edge point
- $x_c = x + x'$
- $y_c = y + y'$

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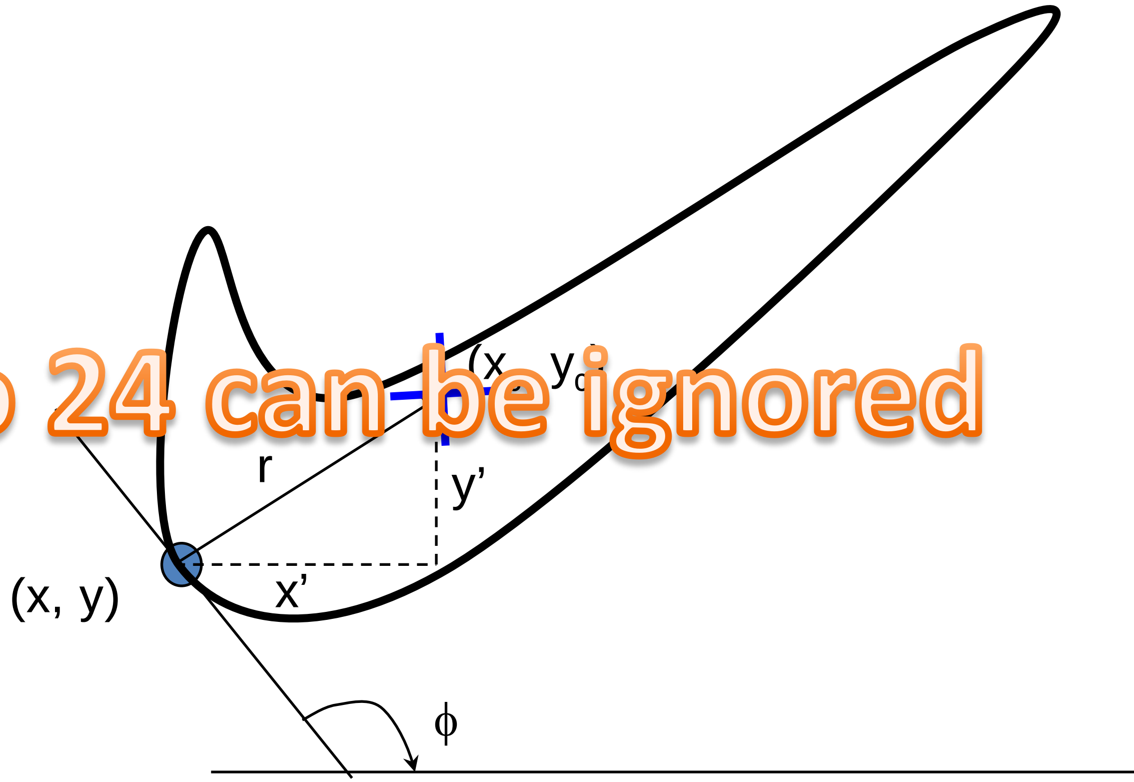

- ϕ is the angle tangent at (x,y) makes with x-axis

GHT - Training

- R-Table is indexed by ϕ

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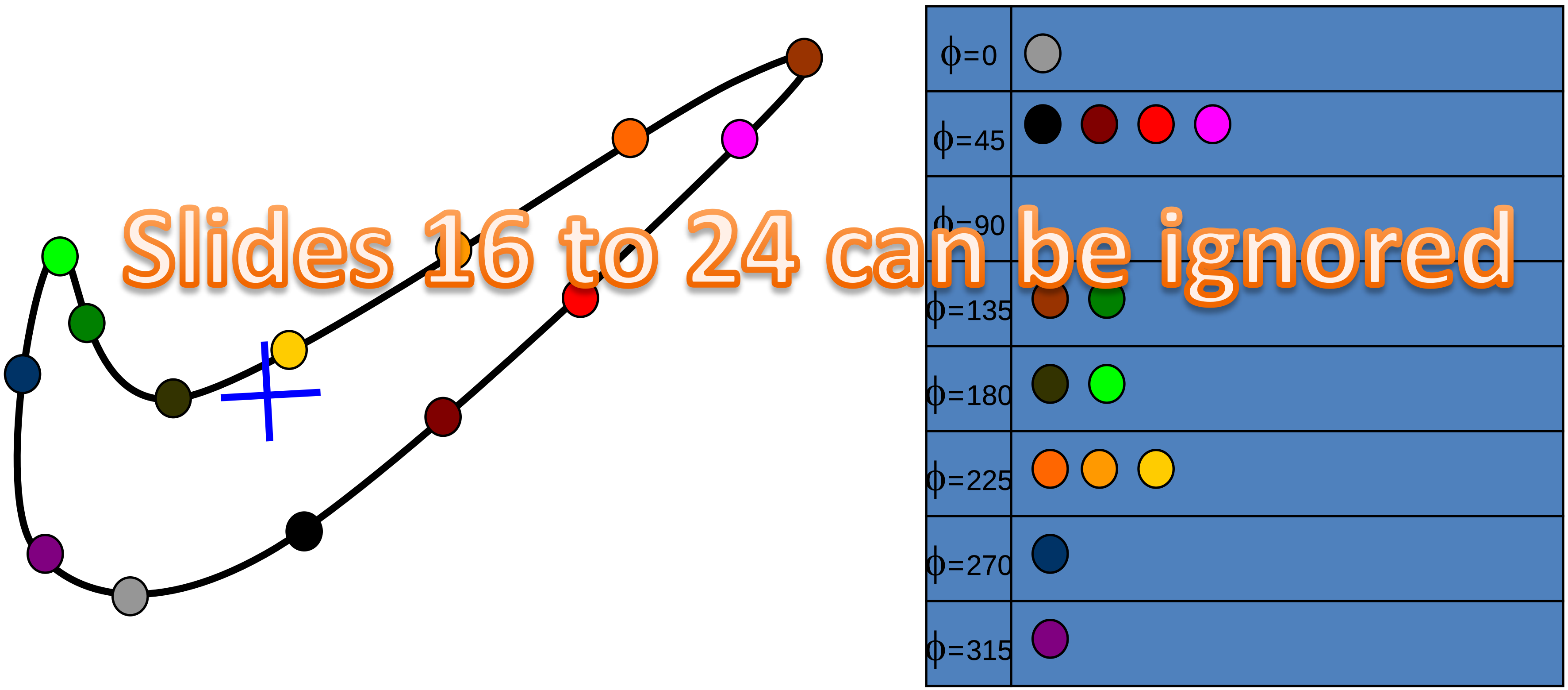
ϕ_1	$r_1^1, r_2^1, r_3^1, \ldots, r_{m_1}^1$
ϕ_2	$r_1^2, r_2^2, r_3^2, \ldots, r_{m_2}^2$
ϕ_3	$r_1^3, r_2^3, r_3^3, \ldots, r_{m_3}^3$
•	•
• •	• •
ϕ_n	$r_1^n, r_2^n, r_3^n, \ldots, r_{m_n}^n$

Example - Training



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Detection

Go to each (x,y) in image

Find φ

For corresponding entry in R Table

Find all possible locations of centroid

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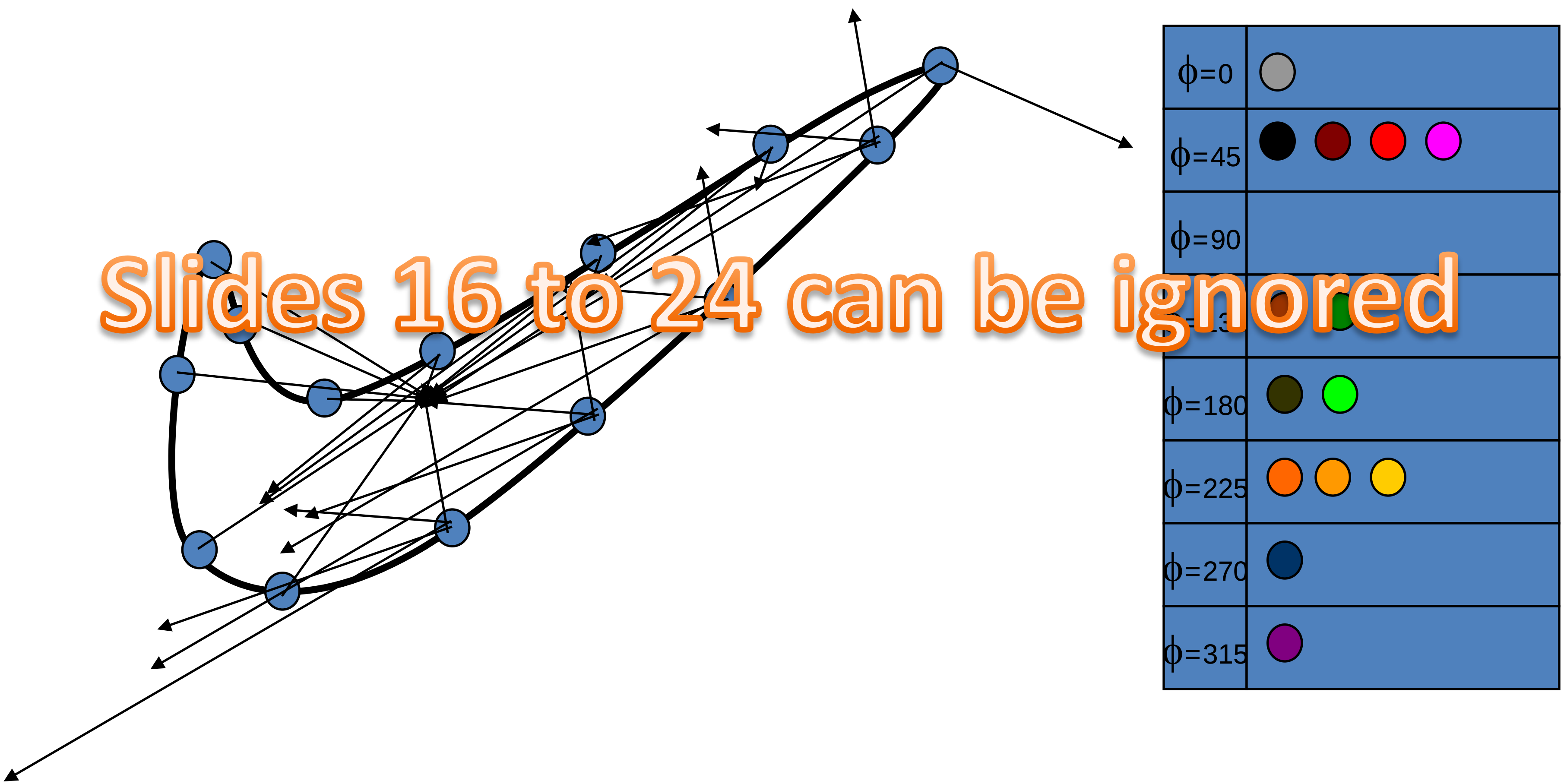
xc = x + x’

yc = y + y’

Increment centroid accumulator by 1

Detection



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Novel Applications

- The concept of voting is a powerful idea that can be applied for other tasks.
- Example: Action Recognition
  - Yao, Angela, Juergen Gall, and Luc Van Gool. "A hough transform-based voting framework for action recognition." *Computer Vision and Pattern Recognition (CVPR), 2010 IEEE Conference on*. IEEE, 2010.
  - "learn the mapping between a 3D video patch and its vote in a 4D Hough space to obtain the class label and the spatiotemporal location of an action in the sense of a generalized Hough transform"

Slides 16 to 24 can be ignored