CS 565 Computer Vision – Assignment 2

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Due Date: Wednesday, 18th November, 2015 before class.

Fourier Transform Theoretical

Please refer to Appendix A on page 4 for help on this part.

1. (**2 marks**) For a complex number $z = a + bi$, compute $z * z$ and $z * \bar{z}$. Which one yields the squared norm $|z|^2 = a^2 + b^2$?

2. (**2 marks**) Verify the relationship $\theta = \omega t$ between angular distance θ, angular speed ω and time t.

3. (**2 marks**) Verify the relationship $\omega = 2\pi f$ between angular speed ω and angular frequency f.

4. (**4 marks**) For a complex vector $\mathbf{f} = (f_1, \dots, f_N)$, compute $\mathbf{f}^T\mathbf{f}$ and $\mathbf{f}^T\bar{\mathbf{f}}$. Which one yields the squared norm of of vector $\mathbf{f}$ (given by $|\mathbf{f}|^2 = |f_1|^2 + \cdots + |f_N|^2$)?

5. (**8 marks**) Prove orthonormality of the Fourier basis. That is, given any two basis vectors $\mathbf{f}_p, \mathbf{f}_q$, prove that

$$\mathbf{f}_p^T\bar{\mathbf{f}}_q = \begin{cases} 1 & p = q \\ 0 & p \neq q \end{cases} \tag{1}$$

   Considering that the Fourier basis is orthonormal, do the different frequencies interfere with each other in representing the signal?

6. Let $\mathbf{x} = (6, 5, 4, 1)^T$ be a signal with 4 values.

   (a) (**2 marks**) Compute the 4 discrete Fourier basis vectors $\mathbf{f}_0, \mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3$.

   (b) (**2 marks**) Compute the coefficients of $\mathbf{x}$ in the discrete Fourier basis.

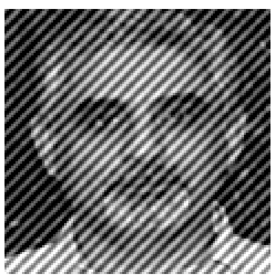
   (c) (**2 marks**) Compute the reconstruction of $\mathbf{x}$ from these coefficients. Is it equal to the original signal $\mathbf{x}$?

Fourier Transform Programming

1. (**2 marks**) In the function **myAffineRescaling.m** add the code that performs an Affine Grayscale Transformation of the input image between 0 and c.

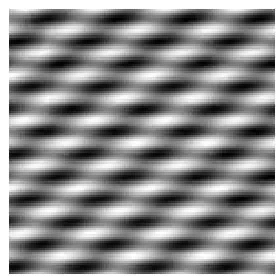
2. (**2 marks**) In the function **myLogDynamicCompression.m** add the code that performs Logarithmic Dynamic Compression of the input image between 0 and c.

3. (**3 marks**) My boss has been abducted and jailed in a funny looking jail as shown in **bossJailed1.png**. I have heard that a strange space exists where getting my boss out of jail is very easy. I have been able to obtain some MATLAB code for going to that strange space and then coming back. This code is present in **unjail_manual.m** but not completely. Can you help me free my boss in **bossJailed1.png** by completing the code in **unjail_manual.m** and then removing the jail bars? Store the result in **bossFreed1.png**.



bossJailed1.png

bossJailed2.png



mystery.png

4. (**3 marks**) The abducters might have placed my boss in a maximum security prison as shown in **bossJailed2.png**. Can you free him from there too? Store the result in **bossFreed2.png**.

5. (**5 marks**) Impressed by your abilities to free captives, the abductors have challenged you to free a mystery captive in **mystery.png**. Can you free this captive as well? Who is the captive? Store the result in **Captive'sLastNameFreed.png**. You may use the code in the file **unjail_manual.m** or **unjail.m** as you see fit.

Submission

Paste your submission as a .zip file into the following folder on \\printsrv:

\\printsrv\Teacher Data\Dr.Nazar Khan\Teaching\Fall2015\CS 565 Computer Vision\Submissions\Assignment2

Write access to this folder will be disabled after the submission deadline. The .zip file should have the following naming convention

`RollNumber_Assignment2.zip`

For example, if your roll number is MSCSF15M999, then the .zip file should be named

`MSCSF15M999_Assignment2.zip`

The .zip file should contain the following directories:

- **Theoretical**
- **Programming**

The **Theoretical** directory should contain the following:

1. A .txt/doc/pdf file called README.txt/doc/pdf containing your answers. If you want to write the answers by hand, then a digital photograph or scanned copy of your answers should be placed here.

The **Programming** directory should contain the following:

1. The files
   (a) **myAffineRescaling.m**
   (b) **myLogDynamicCompression.m**
   (c) **unjail_manual.m**
   (d) **unjail.m**

   supplemented with the missing code.
2. The images
   (a) **bossFreed1.png**,
   (b) **bossFreed2.png**, and
   (c) **Captive'sLastNameFreed.png**.

3. A .txt file called README.txt describing how you managed to free all the captives. This should also include the identity of the mystery captive.

Please do not submit a very large .zip containing extra files. It should only contain what is asked for. If your .zip file contains any extra file(s), you will receive 0 credit for the whole assignment.

Note: To submit your results in a single, beautiful looking .pdf file, the LaTeX source for this document is also provided in the Assignment2.tex file. You can use the command \answer{} to fill in your answers below each question. Please consult your instructor or TA for more help. **Remember: Word is ugly and LaTeX is beautiful!**

A 1D Discrete Fourier Transform

The 1D discrete Fourier transform (DFT) of a **finite**, **sampled** signal $\mathbf{x} = (x_0, \ldots, x_{M-1})^T$ with finite extent is given by

$$\hat{x}_u = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} x_m e^{-i2\pi u \frac{m}{M}} \tag{2}$$

for frequencies $u = (0, \ldots, M-1)$. A signal with M values is decomposed into M frequency coefficients. The corresponding 1D inverse discrete Fourier transform is given by

$$x_m = \frac{1}{\sqrt{M}} \sum_{u=0}^{M-1} \hat{x}_u e^{i2\pi u \frac{m}{M}} \tag{3}$$

for $m = (0, \ldots, M-1)$.

A.1 Interpretation as change of basis

The Fourier basis is given by

$$\mathbf{f}_u = \frac{1}{\sqrt{M}} (e^{i2\pi u \frac{0}{M}}, e^{i2\pi u \frac{1}{M}}, \ldots, e^{i2\pi u \frac{M-1}{M}})^T \tag{4}$$

for frequencies $u = (0, \ldots, M-1)$. A signal $\mathbf{x}$ can be projected onto a basis vector $\mathbf{f}$ via the inner-product

$$< \mathbf{x}, \mathbf{f} >= \sum_{m=0}^{M-1} x_m \bar{f}_m \tag{5}$$

The DFT computes the Fourier basis coefficients $\hat{x}_u =< \mathbf{x}, \mathbf{f}_u >$ for frequencies $u = (0, \ldots, M-1)$. The inverse DFT reconstructs the signal from the Fourier basis coefficients via $\mathbf{x} = \sum_{u=0}^{M-1} \hat{x}_u \mathbf{f}_u$.

A.2 Proving orthogonality of Fourier basis vectors

You might need the formula for the sum of a geometric series

$$\sum_{m=0}^{M-1} r^m = \frac{1 - r^M}{1 - r} \tag{6}$$