CS 565 Computer Vision – Assignment 6

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Due Date: Friday, 15th January, 2016 before 5:30 pm.

1. **(5 marks): Lines and Points in Homogeneous Coordinates**

   For a point $\boldsymbol{x} = (x_1, x_2)^T$ in Euclidean coordinates, the point $\tilde{\boldsymbol{x}} = (x_1, x_2, 1)^T$ is its counterpart in homogeneous coordinates. For any $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^3$ or $\mathbb{P}^2$, the operator $\times$ denotes the cross product

   $$\boldsymbol{x} \times \boldsymbol{y} := \begin{bmatrix} x_2y_3 - x_3y_2 \\ x_3y_1 - x_1y_3 \\ x_1y_2 - x_2y_1 \end{bmatrix}$$

   The cross product $\boldsymbol{x} \times \boldsymbol{y}$ is another vector that is orthogonal to both $\boldsymbol{x}$ and $\boldsymbol{y}$.

   Let $\boldsymbol{m}_1 = (x_1, y_1)^T$ and $\boldsymbol{m}_2 = (x_2, y_2)^T$ be two different points (i.e. $\boldsymbol{m}_1 \neq \boldsymbol{m}_2$) in Euclidean coordinates, and let $\boldsymbol{\ell}_1 = (a_1, b_1, c_1)^T$ and $\boldsymbol{\ell}_2 = (a_2, b_2, c_2)^T$ be non-parallel lines (e.g. $\boldsymbol{\ell}_1$ is defined by $a_1x + b_1y + c_1 = 0$).

   (a) Show that $\boldsymbol{m}_1$ lies on $\boldsymbol{\ell}_1$ if and only if $\tilde{\boldsymbol{m}}_1^T \boldsymbol{\ell}_1 = 0$.

   (b) Let $\boldsymbol{m}_1$ be the intersection of $\boldsymbol{\ell}_1$ and $\boldsymbol{\ell}_2$. Show that $\boldsymbol{\ell}_1 \times \boldsymbol{\ell}_2 = \tilde{\boldsymbol{m}}_1$ holds. (**Hint**: Use the fact that $\boldsymbol{m}_1$ lies on both $\boldsymbol{\ell}_1$ and $\boldsymbol{\ell}_2$.)

   (c) Let $\boldsymbol{\ell}_1$ be the line that connects $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$. Show that $\tilde{\boldsymbol{m}}_1 \times \tilde{\boldsymbol{m}}_2 = \boldsymbol{\ell}_1$. (**Hint**: Use the fact that $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$ both lie on $\boldsymbol{\ell}_1$.)

2. **(4 marks): Transformation Matrices**

   Compute a transformation matrix in homogeneous coordinates which describes a rotation around the y-axis through an angle of 45 degrees followed by a translation with a vector $(-1, 2, -3)^T$ and a rotation around the x-axis through an angle of -60 degrees.

3. **(6 marks): Stereo Reconstruction** Let us assume that we have two cameras in $\boldsymbol{C}_1$ and $\boldsymbol{C}_2$. The extrinsic and intrinsic parameters of both cameras are given in the form of the matrices

   $$\boldsymbol{A}_1^{\textbf{int}} = \boldsymbol{A}_2^{\textbf{int}} = \begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

   $$\boldsymbol{A}_1^{\textbf{ext}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \boldsymbol{A}_2^{\textbf{ext}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & -5 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

   Moreover, the focal length of both cameras is given by $f_1 = f_2 = 1$. Let us assume, that we have found the following correspondence:

   $$\boldsymbol{x}_1 = \left(\frac{21}{2}, \frac{1}{2}\right)^T, \qquad \boldsymbol{x}_2 = \left(\frac{19}{2}, \frac{\sqrt{2}}{2}\right)^T.$$

Here, $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ denote a point in pixel locations in the first and in the second frame, respectively. In order to restore the depth of the original scene point, perform the following steps:

(a) Compute the corresponding optical rays in the 3-D coordinate system of the cameras.

(b) Compute the corresponding optical rays in the 3-D Euclidean world coordinate system.

(c) Intersect these rays to recover the 3-D world coordinates of the 3-D point that was projected on both image planes.

Submission

Submit your hand-written assignment in the instructor's office.