CS-565 Computer Vision

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Lecture 5: Spatial Filtering

Convolution

- Convolution implies
  - Spatial filtering
- What is a filter?
  - Something that lets some things pass through and prevents the rest from passing through.
    - Oil filter, Air filter, Noise filter

Convolution

- We have seen in Lecture 3 that convolution with an averaging mask yields
  - a smooth version of the input signal
  - by suppressing sharp changes (noise)
- The mask is also called a **filter**. **Why?**
- Accordingly, convolution is also called **filtering**.
- Convolution with other masks/filters can yield different results
  - Derivative filtering for edge detection.

Applying Masks to Images

- Convolution Operation
- Mask
  - Set of pixel positions and weights
  - Origin of mask

1	1	1
1	**1**	1
1	1	1

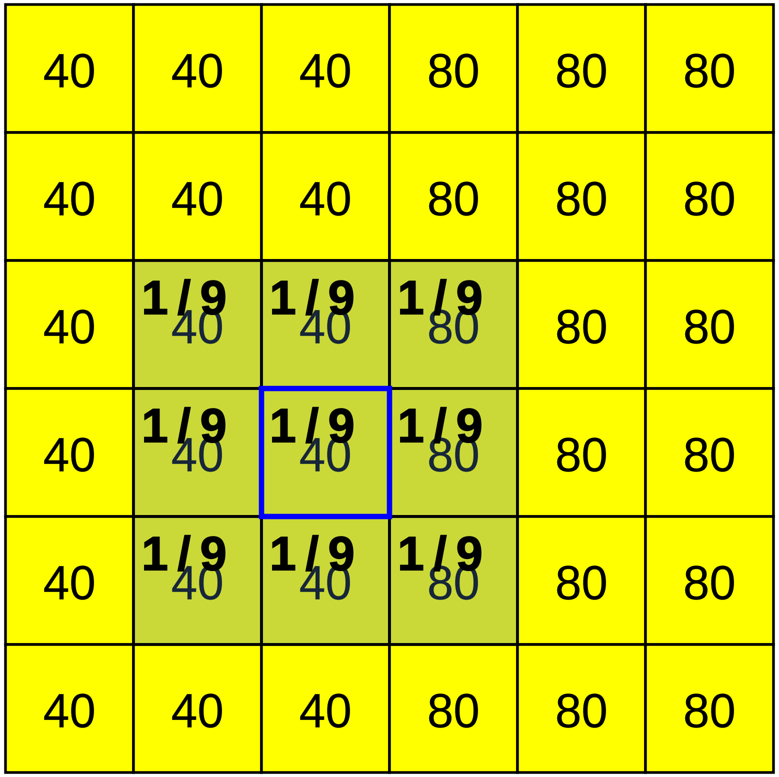
1	2	1
2	**4**	2
1	2	1

1
1
1
1
1

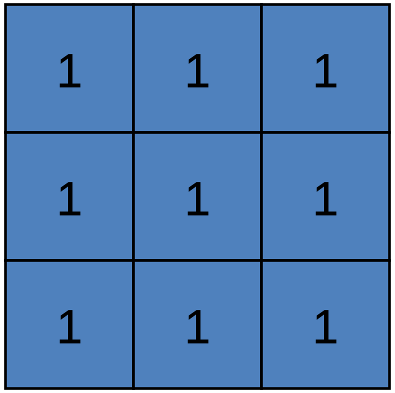
Applying Masks to Images

- $I_1 \otimes$ mask = I_2
- Convention: I_2 is the same size as I_1
- Mask Application:
  - First flip the mask in both dimensions
  - For each pixel *p*
    - Place mask origin on top of pixel
    - Multiply each mask weight with pixel under it
    - Sum the result and put in location of the pixel *p*

Applying Masks to Images



1/9 x



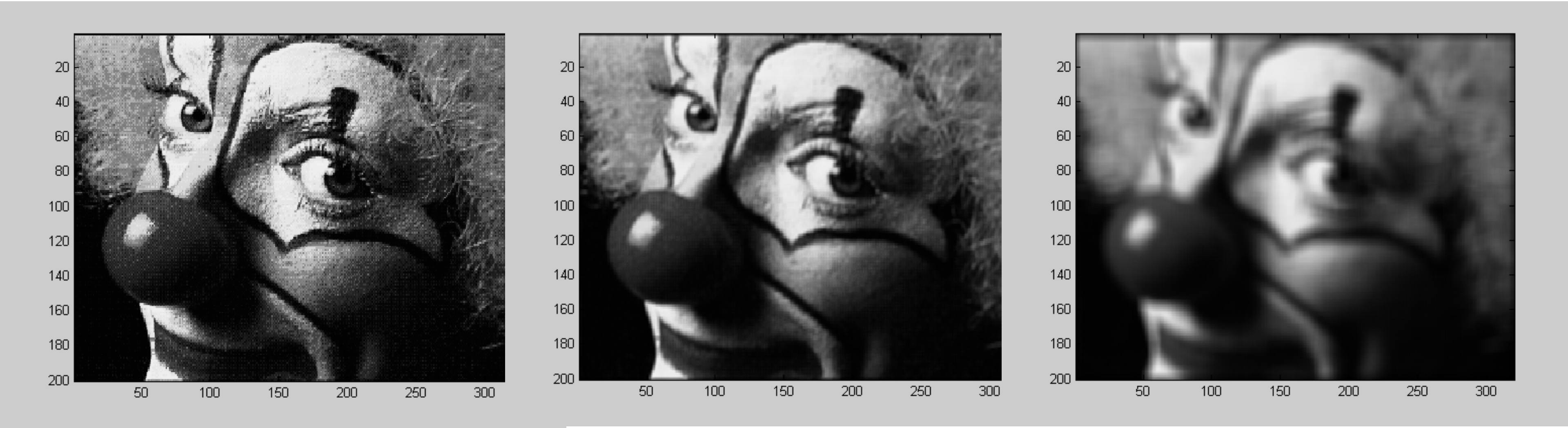
6*(1/9*40)+3*(1/9*80) = 53

Applying Masks to Images

40	40	53	67	80	80
40	40	53	67	80	80
40	40	53	67	80	80
40	40	53	67	80	80
40	40	53	67	80	80
40	40	53	67	80	80

- Overall effect of this mask?
  - Smoothing filter

Applying Masks to Images



Left: Original image. **Middle**: After a convolution with an averaging mask. **Right**: After multiple convolutions with an averaging mask. Author: N. Khan (2014).

What about edge and corner pixels?

- Expand image with virtual pixels
  - Options
    - Fill with a particular value, e.g. zeros
    - Fill with nearest pixel value
    - Mirrored boundary (also called reflecting boundary)
- Fatalism: just ignore them (not recommended)

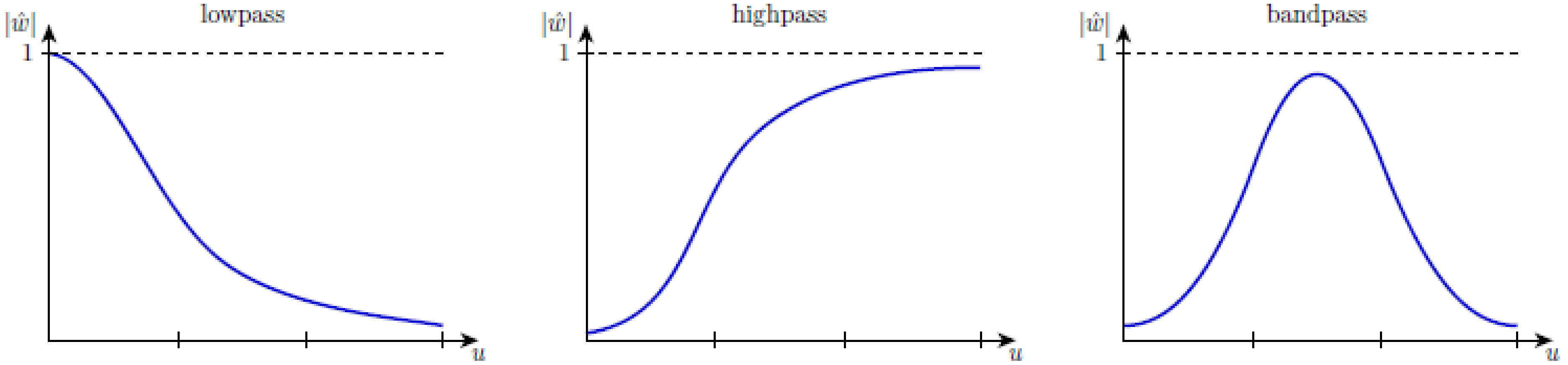
Frequency Interpretation

- Noise is the high frequency component of a signal.
- Convolution with averaging mask is equivalent to reducing the high frequency components of a signal.
- Convolution with Gaussian mask also reduces high frequency components.

Frequency Interpretation

- Lowpass filters
  - Low frequencies are allowed to pass unaffected.
- Highpass filters
  - High frequencies are allowed to pass unaffected.
- Bandpass filters
  - Frequencies within a certain range (band) are allowed to pass unaffected.

Frequency Interpretation



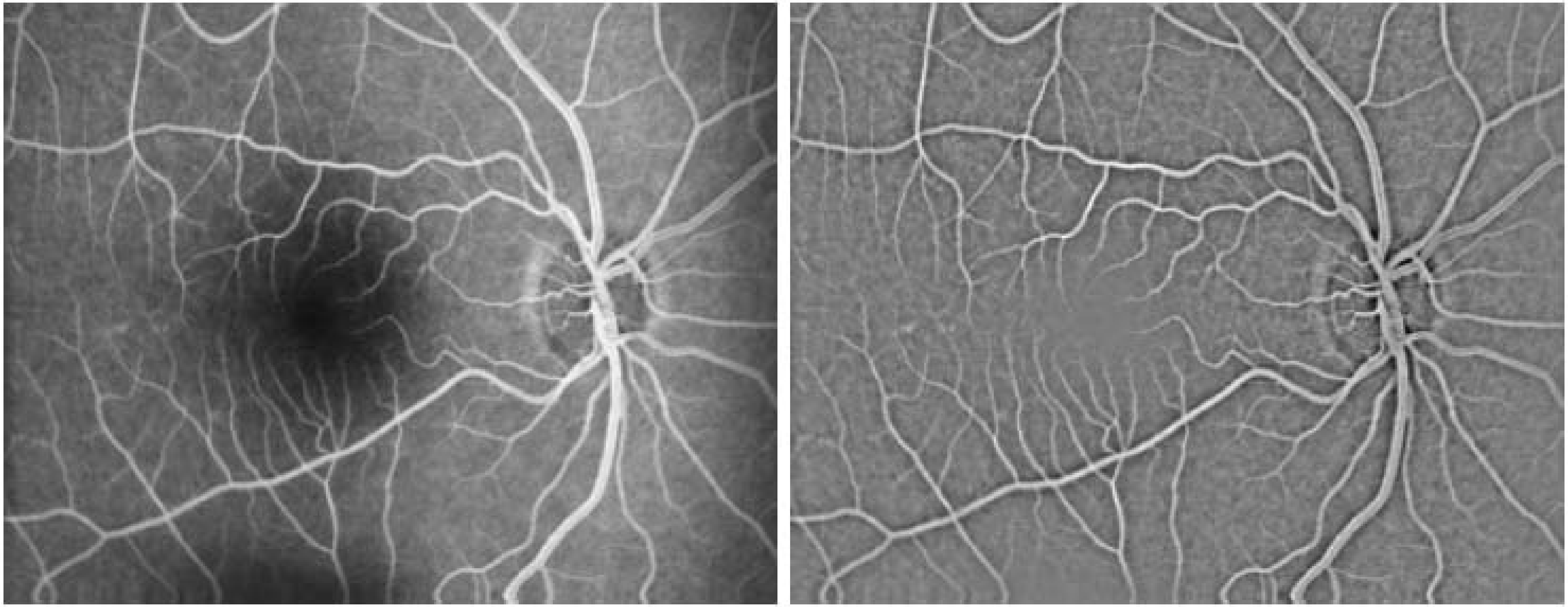
Lowpass:
Reduce high frequencies by giving them less weight.

Highpass:
Reduce low frequencies by giving them less weight.

Bandpass:
Reduce frequencies outside a certain band by giving them less weight.

Highpass Filtering

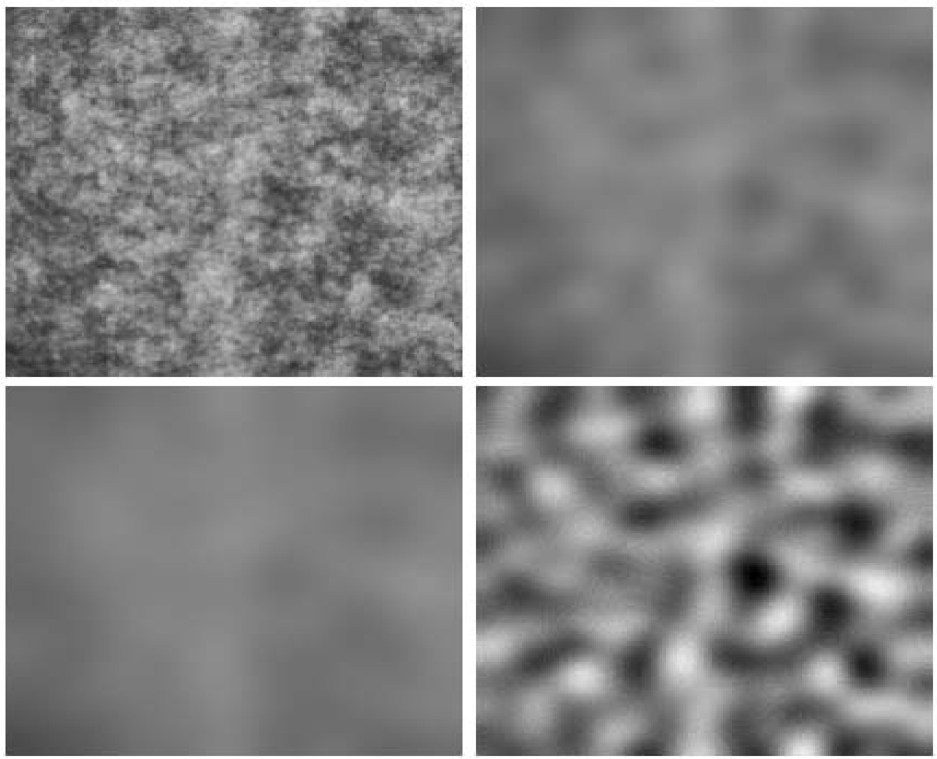
- H=I-Gaussian*I



Left Vessel structure of the background of the eye. **Right:** Elimination of low-frequent background structures by subtracting a Gaussian-smoothed version from the original image. The greyscale range $[-94, 94]$ has been rescaled to $[0, 255]$ by an affine rescaling. Author: J. Weickert (2002).

Bandpass Filtering

- B=G1*I – G2*I



(a) Top left: Fabric, 257×257 pixels. **(b) Top right:** After lowpass filtering with a Gaussian with $\sigma = 10$. **(c) Bottom left:** Lowpass filtering with $\sigma = 15$. **(d) Bottom right:** Subtracting (b) and (c) gives a bandpass filter that visualises cloudiness on a certain scale. The greyscale range has been affinely rescaled from $[-13, 13]$ to $[0, 255]$. Author: J. Weickert (2002).

Some Properties of Convolution

- Commutativity
  - I*M = M*I
  - Signal and kernel play an equal role.
- Associativity
  - $(I*M_1)*M_2 = I*(M_1*M_2)$
  - Successive convolution with kernels M_1 and M_2 is equivalent to a single convolution with kernel M_1*M_2.

Some Properties of Convolution

- Shift invariance
  - Translation(I*M) = Translation(I)*M
  - Translation of convolved signal is equivalent to convolution with translated signal.
- Linearity
  - (aI+bJ)*M = a(I*M) + b(J*M) for all a, b $\in \mathbb{R}$
  - Single convolution of a linear combination of signals is equivalent to a linear combination of multiple convolutions.