CS-565 Computer Vision

Nazar Khan

Lecture 3

A Note on Pre-Requisites

- Pre-requisites
  - Linear Algebra
  - Probability
  - Calculus
- **We will cover some basics as they come along.**
- So don’t worry too much.
- However, it will serve you well to read the Appendices of standard Computer Vision, Image Processing or Computer Graphics books. They are usually **very** helpful
  - Appendix from Rich Szeliski’s book
  - Appendix from Gonzalez & Woods’ book

Study Tip

- These slides are available before class in the course folder.
- Before class:
  - Print them
  - Read them
- During class:
  - Take notes on them
- This will save you LOTS OF effort after class.

Topics to be covered

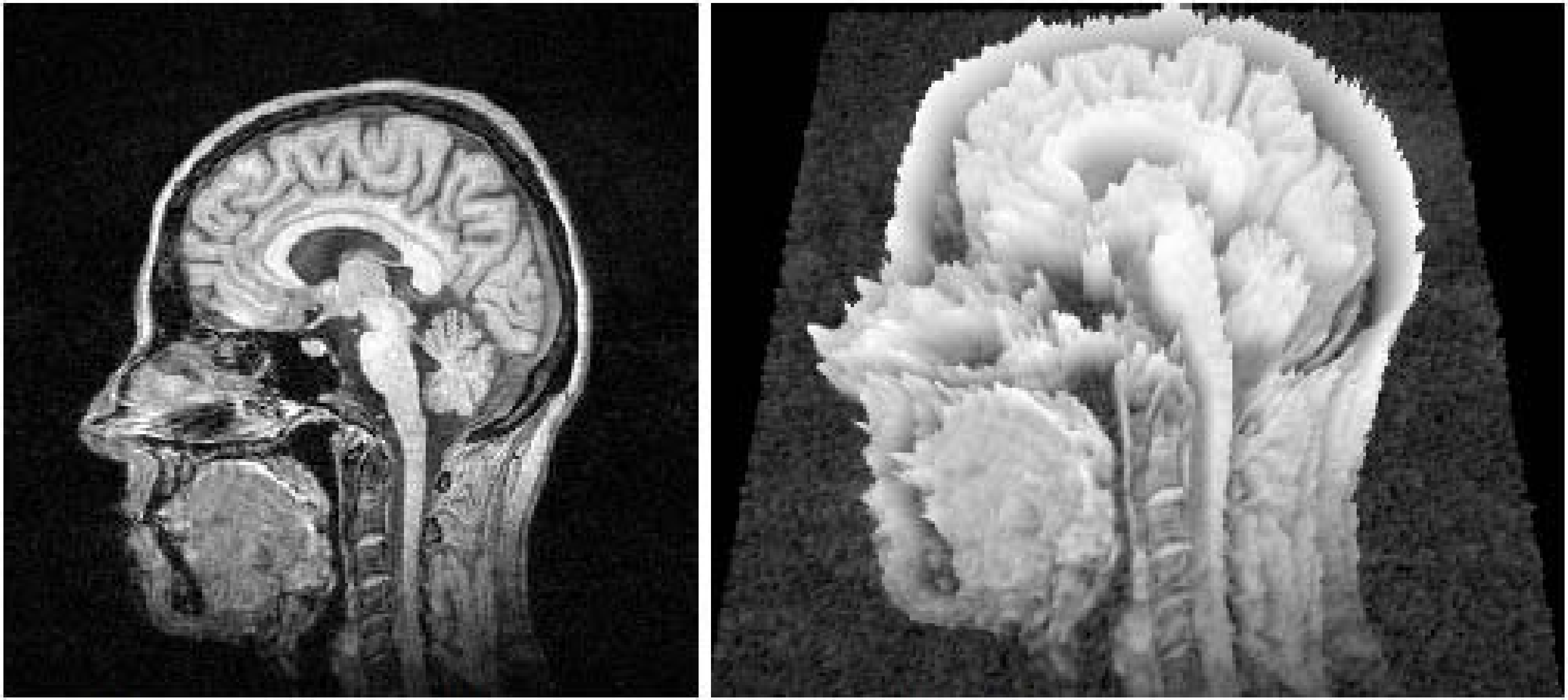
- Image types
- Sampling and Quantization
- Noise models

Image Concepts

- What is a grayscale image?
  - A mapping from a rectangular domain Ω=(0,r) x (0,c) to the range $\mathbb{R}$

$$f : \mathbb{R}^2 \supset \Omega \to \mathbb{R}$$

- The domain is called image domain or image plane
- The range specifies grey value
- Usually low grey values are dark and high grey values bright.



Left: Magnetic resonance (MR) image of a human head. **Right:** Representation as a function $f(x, y)$ over a rectangular image domain Ω. Authors: J. Weickert, C. Schnörr (2000).

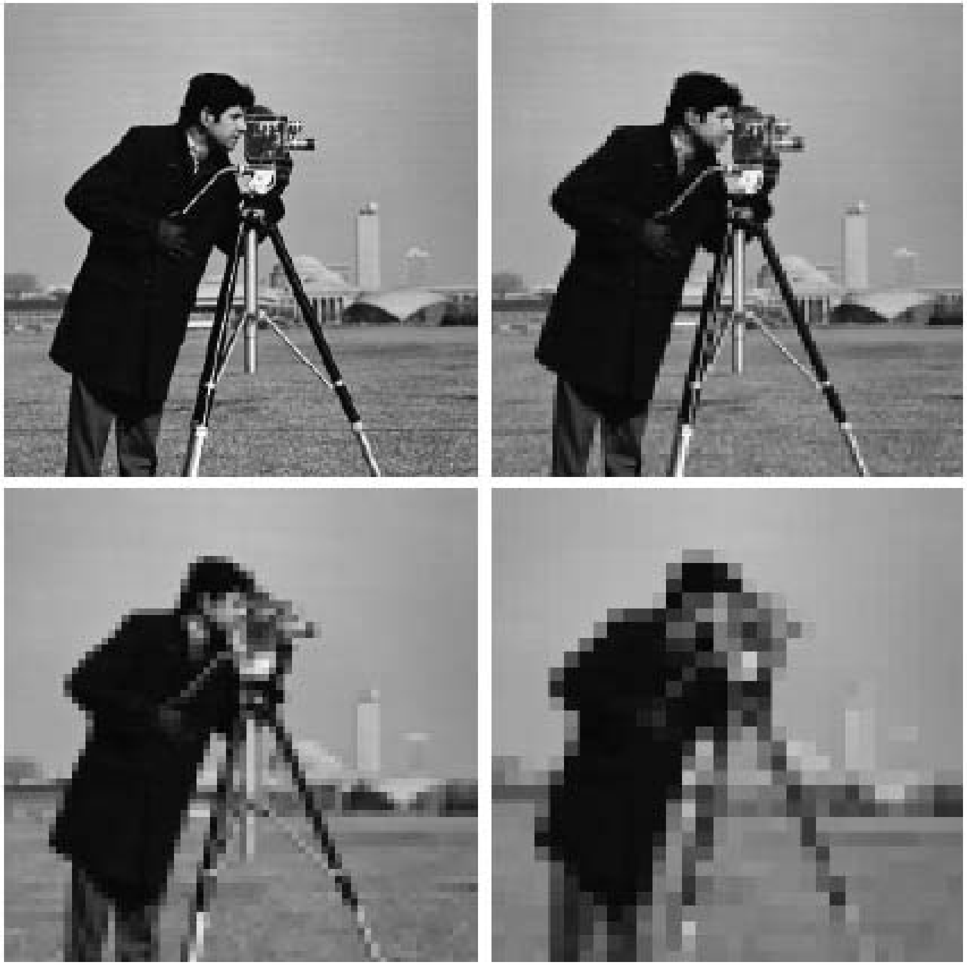
Sampling

- Discretization of the domain Ω
- Image data lie on a rectangular grid of points
- This creates a digital image

$$\{f_{i,j} \mid i=1,\ldots,m;\ j=1,\ldots,n\}$$

- Grid point is called a pixel (picture element)
  - Pixel dimensions are usually the same in both directions.
- Sampling determines image quality

Sampling



Digital test image with different sampling rates. **Top left:** Sampled with 256×256 pixels. **Top right:** 128×128 pixels. **Bottom left:** 64×64 pixels. **Bottom right:** 32×32 pixels. Author: J. Weickert (2000).

Quantization

- Discretization of the range $\mathbb{R}$
- Saves disk space
- If gray value is coded by a single byte, then the discrete range is given by?
  - {0,1,…,255}
- Range of binary images?
  - {0,1}
- Humans can distinguish only 40 grayscales
- But we are also very good at analyzing binary images.

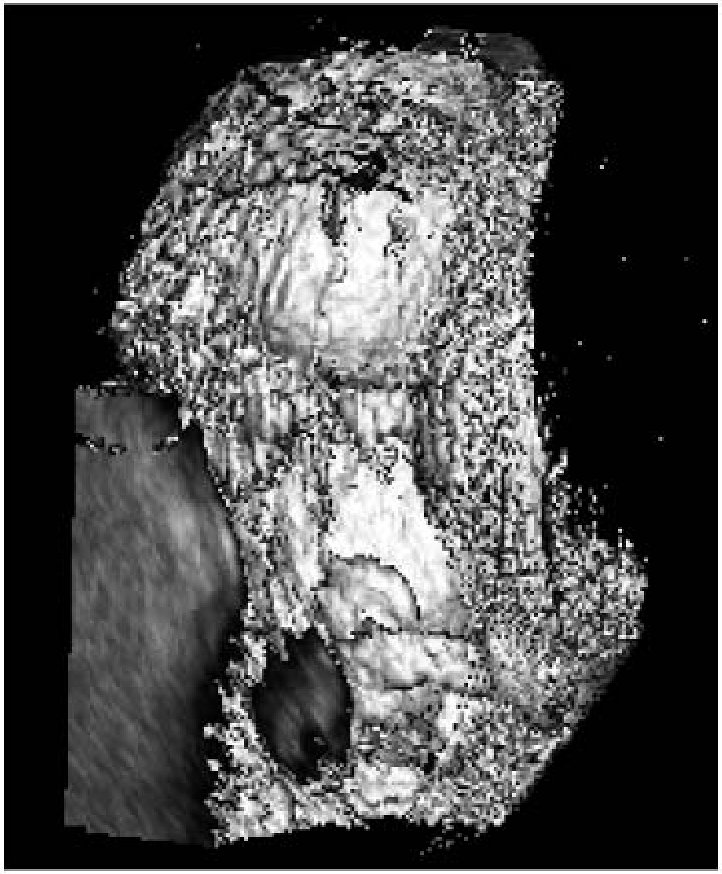
Quantization

Digital test image (256 × 256 pixels) with different quantisation rates. **Top left:** 256 greyscales. **Top right:** 32 greyscales. **Bottom left:** 8 greyscales. **Bottom right:** 2 greyscales. Author: J. Weickert (2000).

Image Types

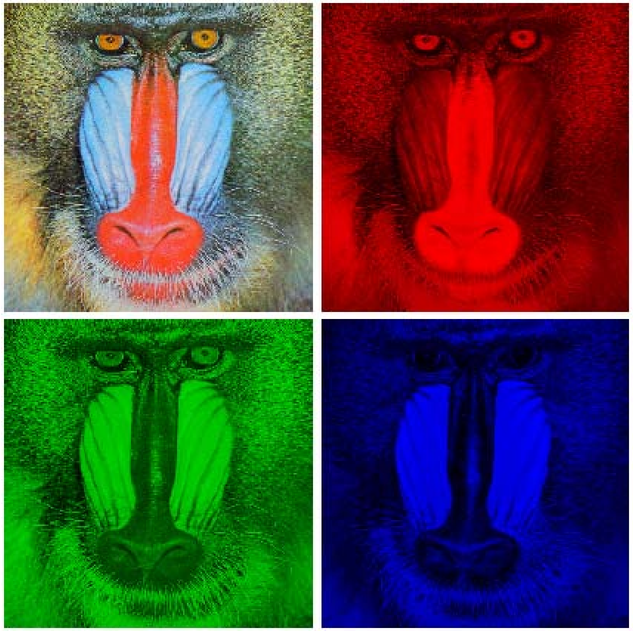
- **m-dimensional images**
- Domain in $\mathbb{R}^m$
- m=1: 1D signals (audio)
- m=2: 2D images
- m=3: 3D images (CT Scan, MRI, Kinect)
  - Image points in 3D are called voxels (volume elements)
  - Voxel dimensions usually differ in different directions.

Image Types



Rendering of a 3-D ultrasound image of a human fetus in its 10th week. Authors: J. Weickert, K. Zuiderveld, B.M. ter Haar Romeny, W. Niessen (1997).

Image Types



- **Vector Valued Images**
- Range in $\mathbb{R}^n$
- Equivalent to having n channels
- Examples:
  - Color Images
    - 3 channels – Red, Green Blue
    - Humans can distinguish 2,000,000 colours!
  - Multispectral images
    - Satellite images
    - Many channels (4-30) that represent different frequency bands.

Image Types

- **Matrix valued images**
- Range in $\mathbb{R}^{n \times n}$
- Every pixel location stores an n-by-n matrix
  - Useful in medical imaging

Image Types

- **Image Sequences**
- Any of the above types of images can be considered in sequence
- Domain will change from $\mathbb{R}^m$ to $\mathbb{R}^{m+1}$.
- For this class, we will mainly be concerned with 2D grayscale images and/or their sequences (videos).

NOISE MODELS

Noise Models

- Noise
  - Additive Noise
  - Multiplicative Noise
  - Impulse Noise
  - Measuring Noise
- Blur
  - Convolutions
  - Modeling Blur by Convolutions
- Combined Blur and Noise

Noise

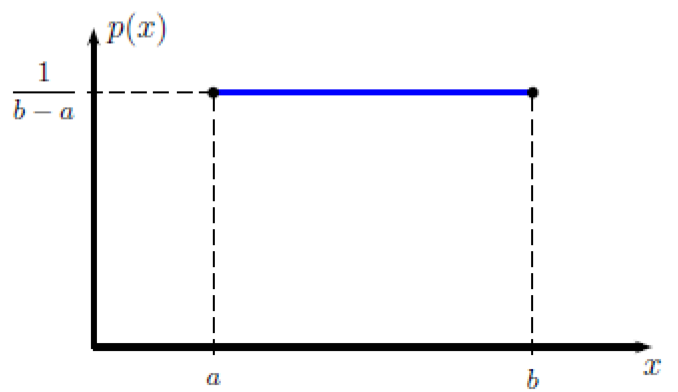
- Very common in digital images (or any real-world data)
- Can have many reasons, e.g.
  - image sensor of a digital camera
  - grainy photographic films that are digitised
  - specific acquisition methods:
    - e.g. ultrasound imaging always creates ellipse-shaped speckle noise
  - atmospheric disturbance during wireless transmission

Additive Noise

- Most important type of noise
  - F=G+N where G is the original image and N is the noise.
- Distribution of N
  - Uniform (pretty easy)
  - Gaussian (pretty common)

Uniform Additive Noise

- Not a very realistic model of noise
- But easy to simulate
- Constant density function between a and b
- F=G+U where every pixel in U is uniformly distributed between a and b



Density function for uniform noise. Author: M. Mainberger (2008).

Uniform Additive Noise

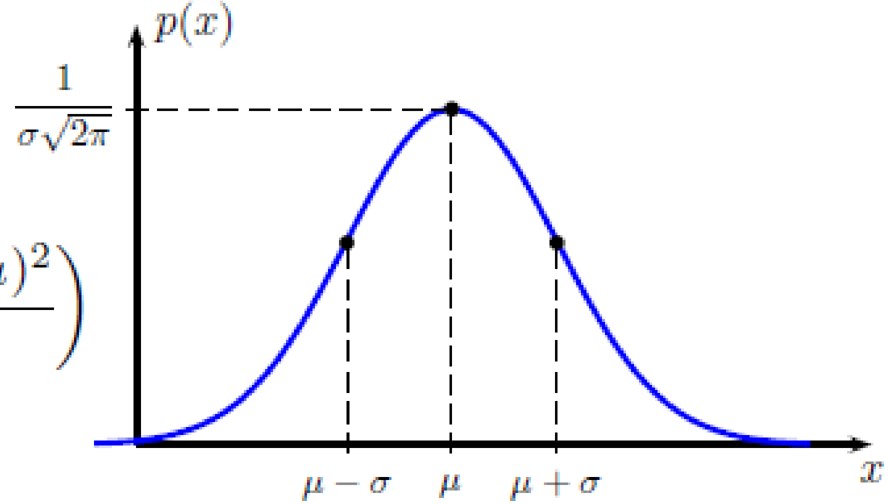
Left: Original image, 256×256 pixels, grey value range: $[0, 255]$. **Right:** After adding noise with uniform distribution in $[-70, 70]$. Resulting grey values outside $[0, 255]$ have been cropped. Author: J. Weickert (2007).

Gaussian Additive Noise

- Most important noise model
  - thermal noise from the image sensor
  - circuit noise from signal amplifications
- When many sources of noise are combined, the cumulative noise can be modeled using a Gaussian density
- F=G+ $\aleph(\mu, \sigma)$

Gaussian Additive Noise

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)$$



Density function for Gaussian noise. Author: M. Mainberger (2008).

- Gaussian noise lies almost completely within the interval $\mu \pm 3\sigma$

Gaussian Additive Noise

Left: Original image, 256×256 pixels, grey value range: $[0, 255]$. **Right:** After adding Gaussian noise with $\sigma = 64.48$. Grey values outside $[0, 255]$ have been cropped. Author: J. Weickert (2002).

Multiplicative Noise

- Signal dependent
  - noise caused by grains of a photographic emulsion
- F=G+N.*G

Multiplicative Noise

Left: Original image, 256×256 pixels, grey value range: $[0, 255]$. **Right:** After applying multiplicative noise where n has uniform distribution in $[-0.5, 0.5]$. Resulting grey values outside $[0, 255]$ have been cropped. Note that darker grey values are less affected by noise than brighter ones. Author: J. Weickert (2007).

Impulse Noise

- Degrades only <u>some</u> pixels.
  - Additive and multiplicative noise affects <u>all</u> pixels
  - Defect in the imaging sensor
- Unipolar – defective pixels have the same wrong gray value
- Bipolar – defective pixels can have either of 2 wrong gray values
  - salt-and-pepper noise – max and min gray value

Impulse Noise

Left: Original image, 256×256 pixels. **Right:** 20 % of all pixels have been degraded by salt-and-pepper noise, where bright and dark values have the same probability. Author: J. Weickert (2002).

Measuring Noise

- Mean Squared Error: $\|F - G\|^2$

$$\mathrm{MSE}(f,g) := \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(f_{i,j} - g_{i,j})^2.$$

  – The smaller the better
- Peak-Signal-to-Noise Ratio:

$$\mathrm{PSNR}(f,g) := 10 \log_{10}\left(\frac{255^2}{\mathrm{MSE}(f,g)}\right)$$

  – The higher the better
  – Unit is decibel (dB)
  – PSNR <30 dB starts to become noticeable

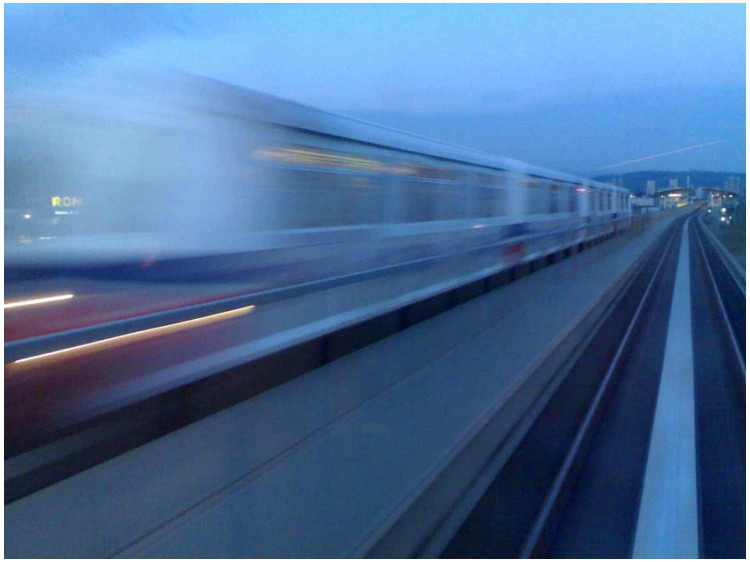
Measuring Noise

Top left: Original image, 256×256 pixels. **Top right:** Adding Gaussian noise with $\sigma = 15$ gives $\mathrm{MSE} = 226.06$ and $\mathrm{PSNR} = 24.59$ dB. **Bottom left:** $\sigma = 30$ yields $\mathrm{MSE} = 904.24$ and $\mathrm{PSNR} = 18.57$ dB. **Bottom right:** $\sigma = 60$ yields $\mathrm{MSE} = 3616.95$ and $\mathrm{PSNR} = 12.55$ dB. Grey values outside $[0, 255]$ are cropped. Author: J. Weickert (2009).

Blur

- Second source of image degradation besides noise
  - Defocusing,
  - Imperfections of the optical system,
  - Motion blur

Blur

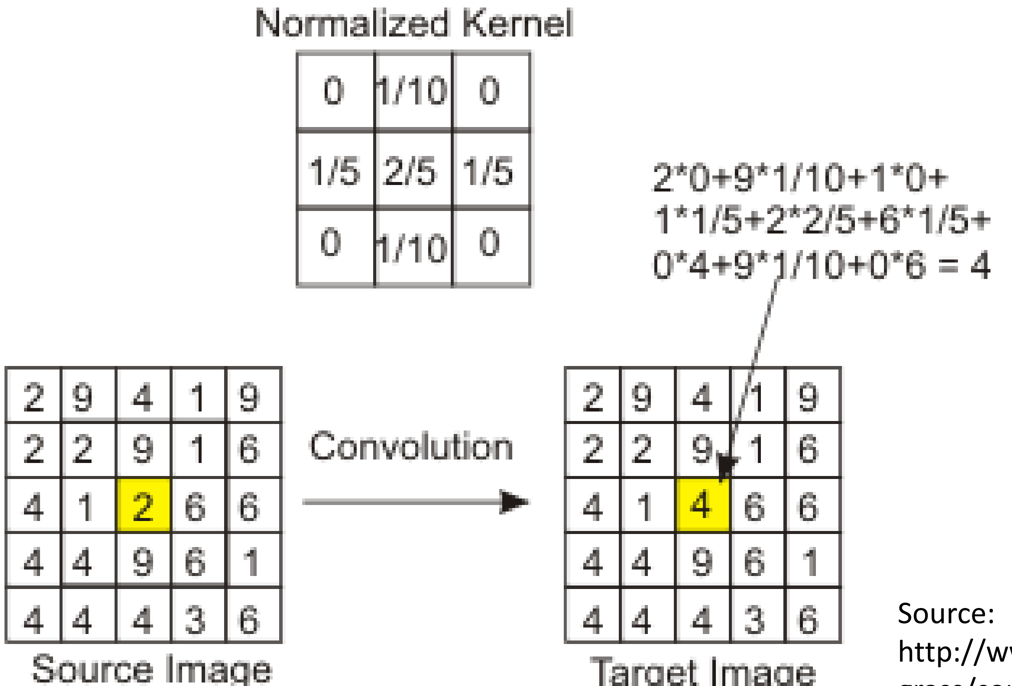



Blur

- Simplest blur – shift invariant (same amount of blurring at all image locations)
- Can be thought of as a weighted averaging within a certain neighbourhood
  - Averaging: $\frac{1}{n}\sum_{i=1}^{n} g_i$
  - Weighted averaging: $\sum_{i=1}^{n} w_i g_i$

Blur

- Moving weighted averaging can be achieved via **convolution**
- For every image pixel
  - Place mask on the image pixel
  - Take dot product of mask and image region under mask
  - Store result on that pixel's location in new image

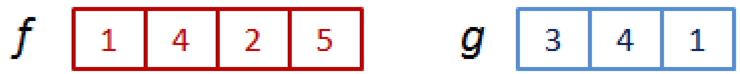


Source: http://www.ahristov.com/taller/procgraph/grass/convolution.gif

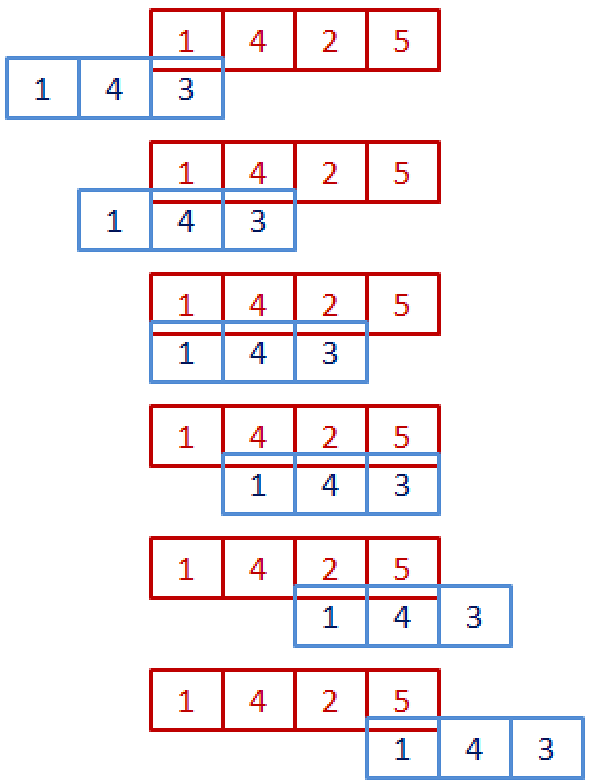
Blur

Source: http://cg2010studio.files.wordpress.com/2012/05/gaussian-smoothing.jpg

Convolution



$$c = f * g$$



C[0] = 1*3 = **3**

C[1] = 1*4 + 4*3 = **16**

C[2] = 1*1 + 4*4 + 2*3 = **23**

C[3] = 4*1 + 2*4 + 5*3 = **27**

C[4] = 2*1 + 5*4 = **22**

C[5] = 5*1 = **5**

http://toto-share.com

Convolution

German stock market index (DAX) on October 20, 2005. **Blue:** Daily values. **Red:** Averaged over the last 38 days. **Green:** Averaged over the last 200 days. Source: `http://www.spiegel.de`.

Convolution

$$(g * w)_i := \sum_{k \in \mathbb{Z}} g_{i-k} w_k$$

- Properties
  - Commutativity: $f * g = g * f.$
  - Associativity: $(f * g) * h = f * (g * h).$
  - Distributivity: $(f + g) * h = f * h + g * h,$
    $f * (g + h) = f * g + f * h.$