CS-565 Computer Vision

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PUCIT

Lecture 10: Corner Detection

Assignment 1

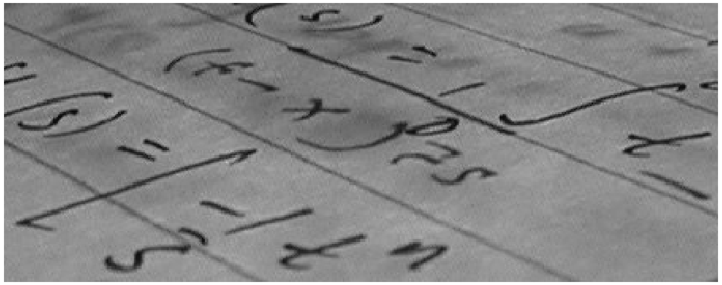
- Graduate students should **know** that cheating is wrong.
  - They do not need to be reminded.
  - Or given second chances.
- Some students resorted to cheating in Assignment 1.
- Their cases will be decided by the Disciplinary Committee.

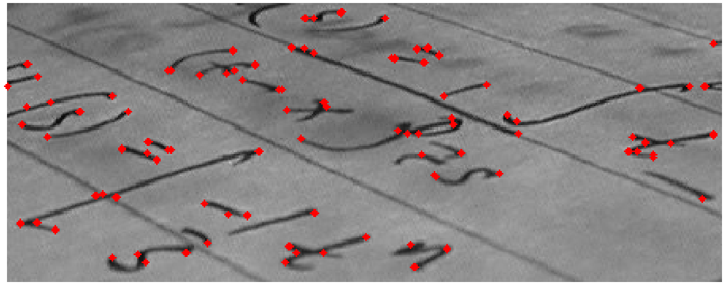
Mid-term

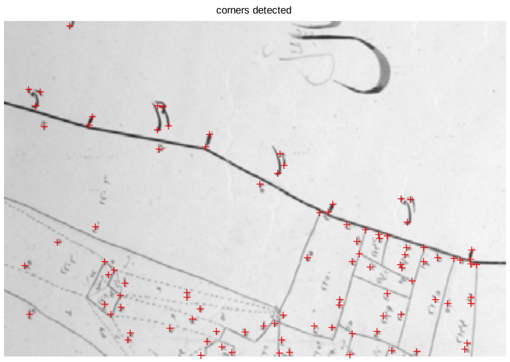
- Please let me know if you need any revision sessions.

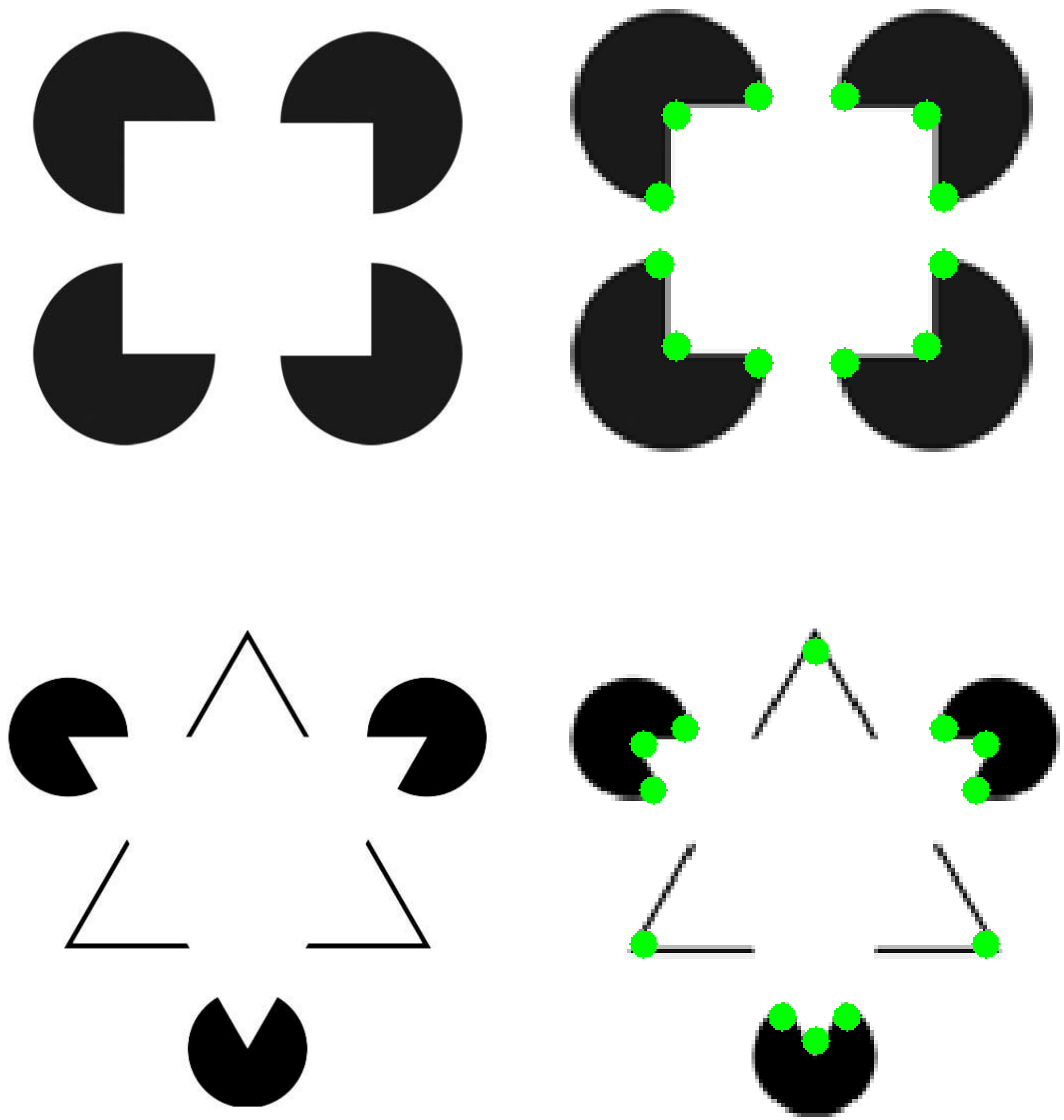
What is a Corner?

- A corner may be interpreted as an intersection of two edges.
- Or as a region where strong derivatives exist in more than one direction.





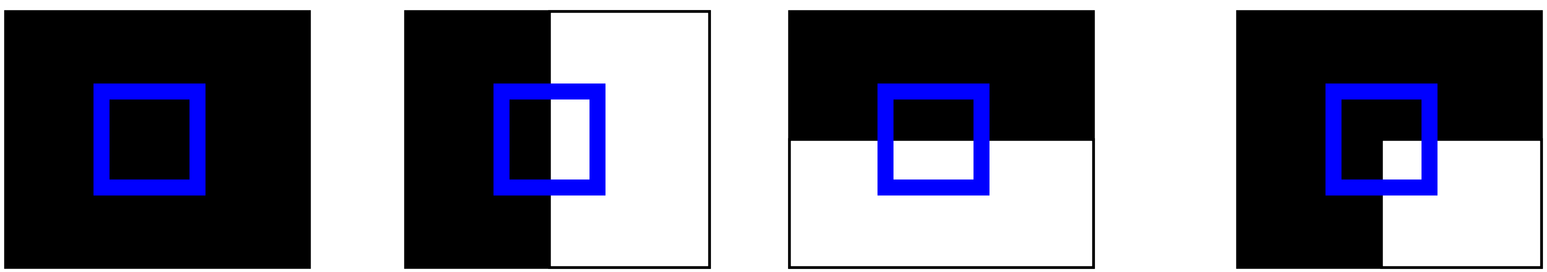




Corners detected using the Harris corner detector. Author Nazar Khan (2014)

Corner Detection

- A corner is different from its surroundings.
- A patch placed on a corner pixel is different from its surrounding patches.

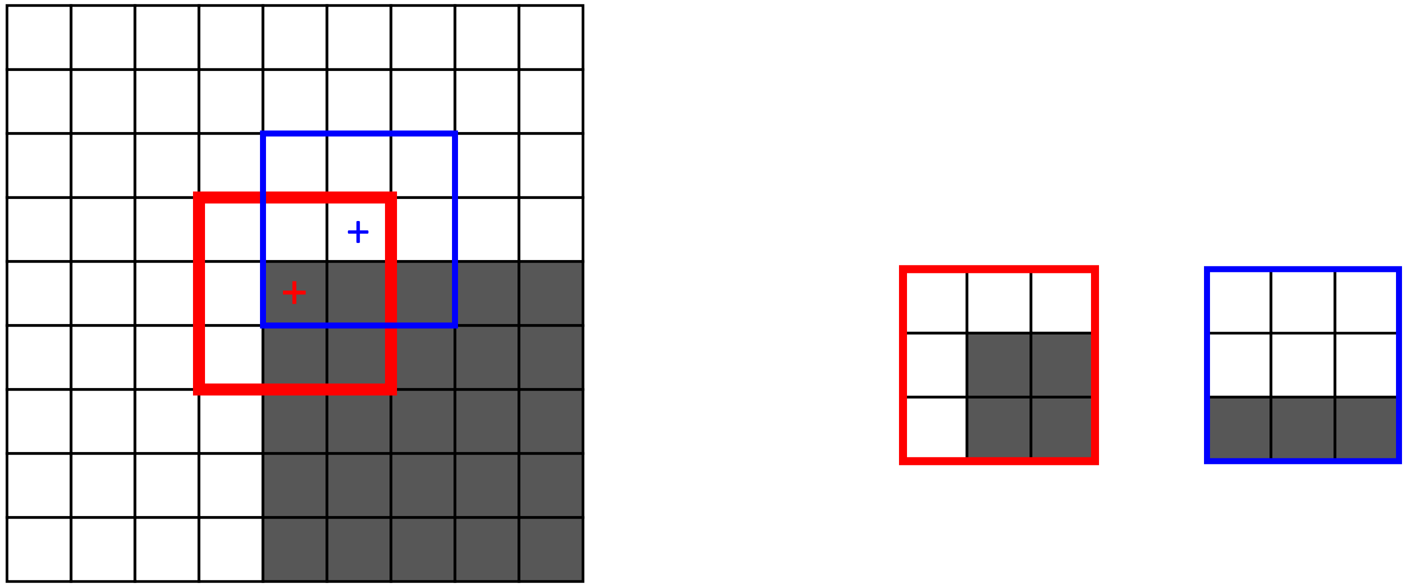


Patch is similar to patches in all directions

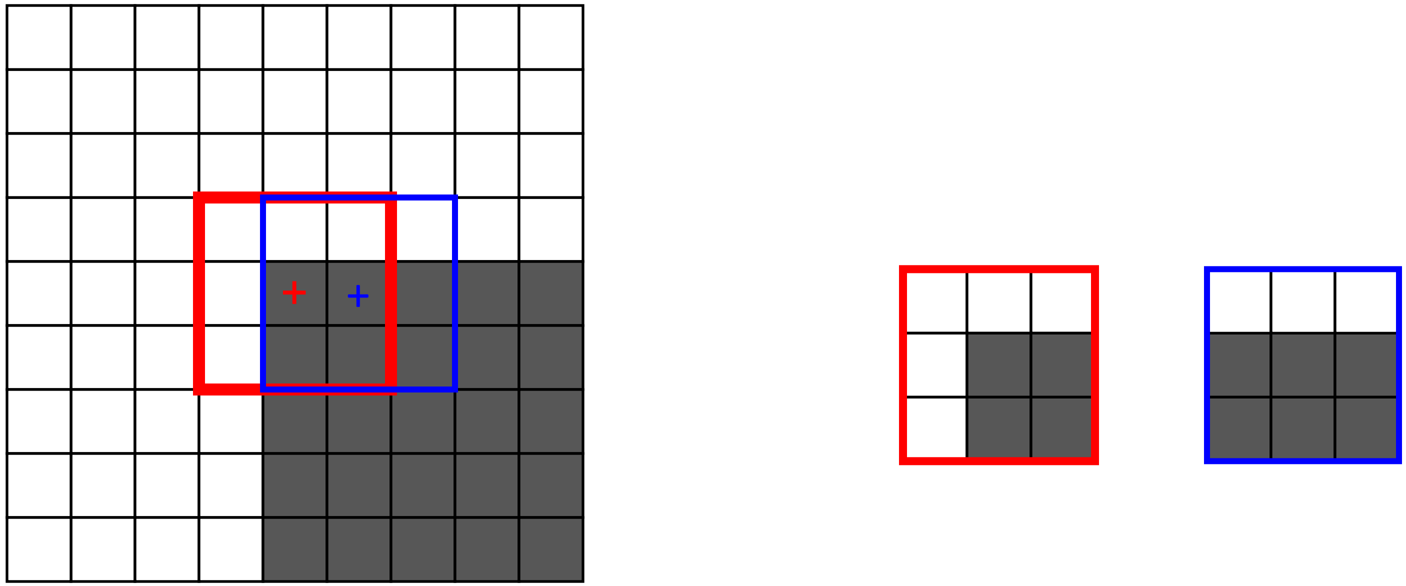
Patch is similar to patches in vertical direction

Patch is similar to patches in horizontal direction

Patch is NOT similar to patches in all directions



Demo of a point + with well distinguished neighbourhood.



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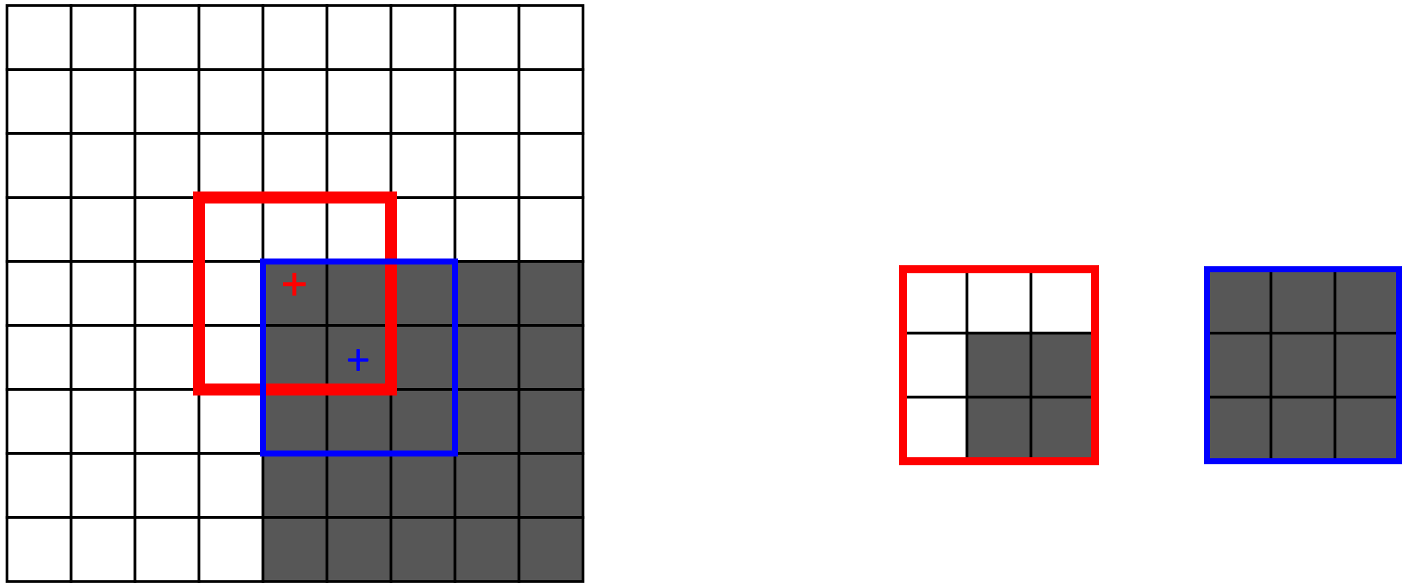

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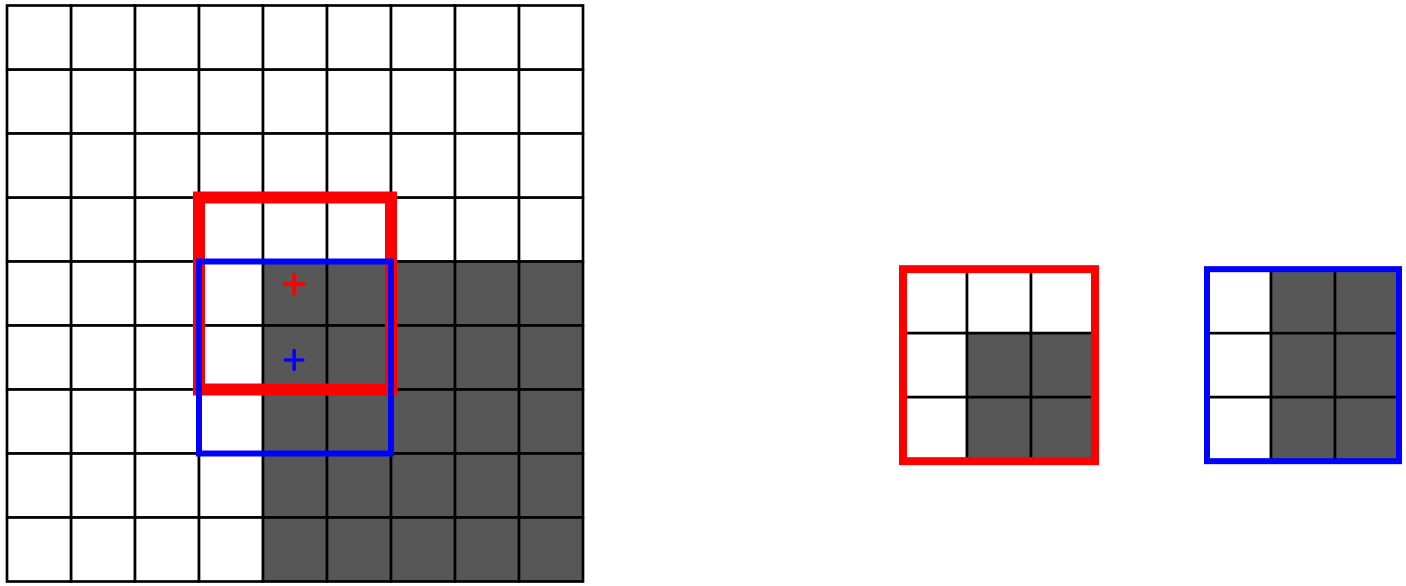

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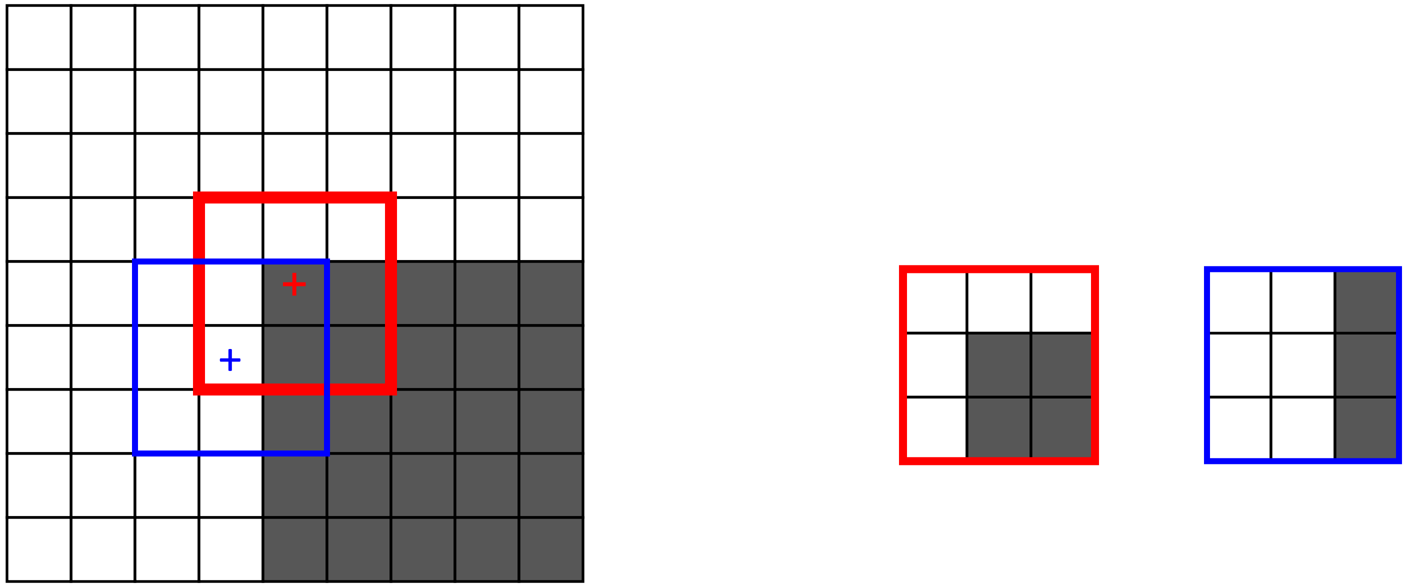
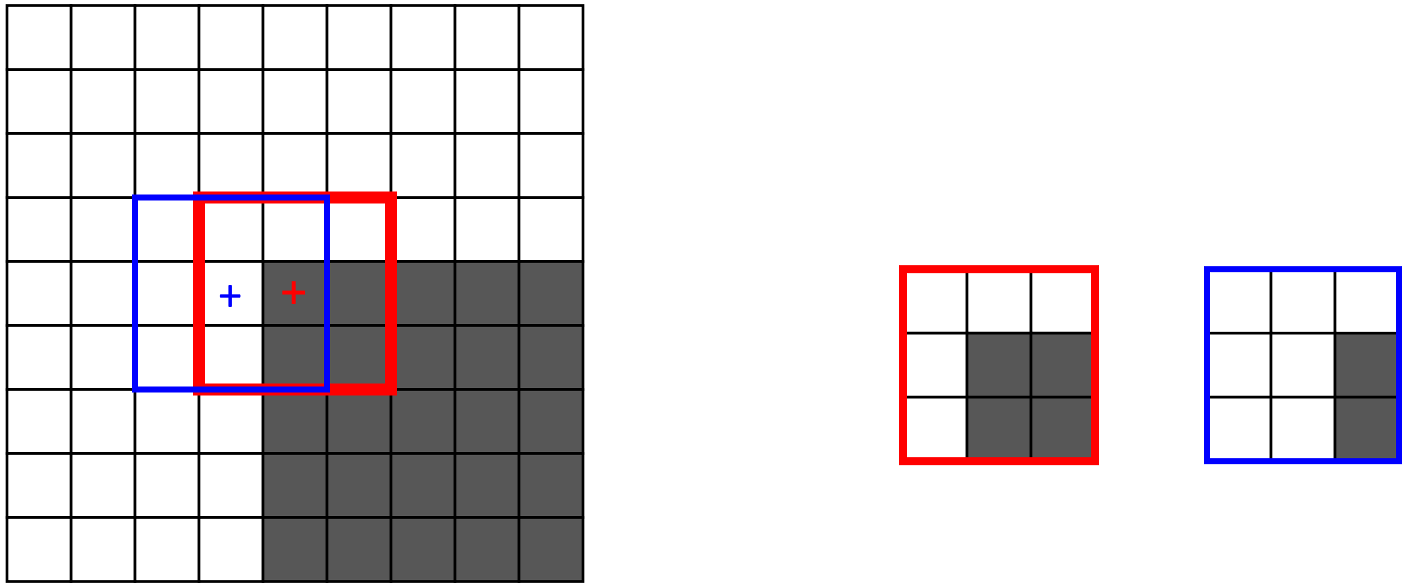
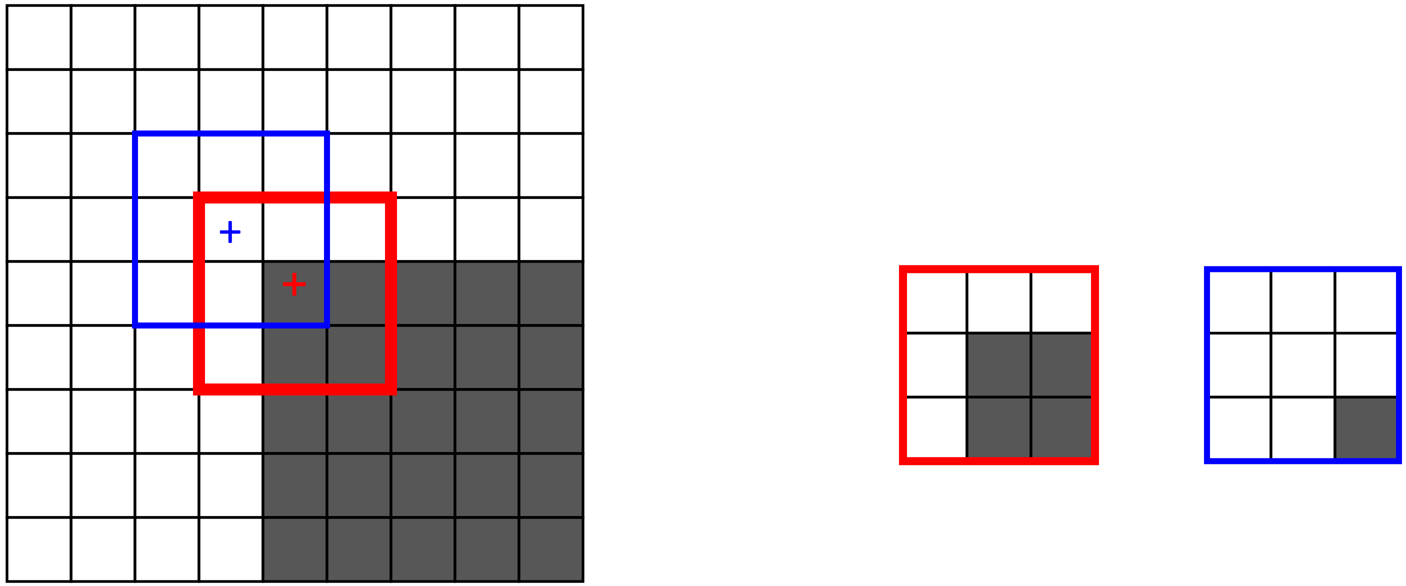


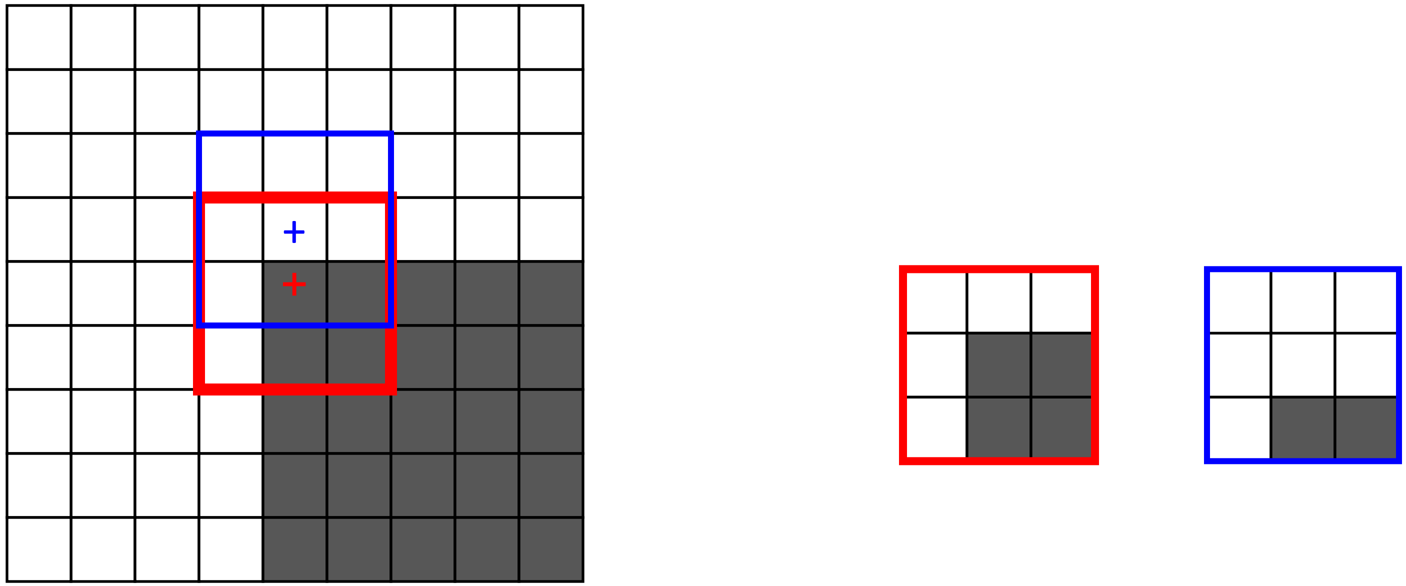
Demo of a point + with well distinguished neighbourhood.



Demo of a point + with well distinguished neighbourhood.



Demo of a point + with well distinguished neighbourhood.



Demo of a point + with well distinguished neighbourhood.

Patch Similarity

- Similarity of two patches p and q can be computed via the sum-of-squared-differences (SSD)

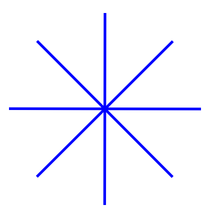
$$\mathrm{SSD}(p,q) = \sum\sum(p_{ij}-q_{ij})^2$$

Moravec Corner Detector

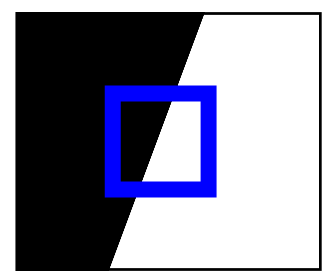
- Among the early corner detection approaches.
- **Basic Idea**: A corner should have low self-similarity.
  - **Self-similarity**: how similar a patch is to neighbouring, overlapping patches.
- **Corner Strength**: min(SSD(p,q_N)) where N is the set of neighbouring, overlapping patches of *p*.
  - If *p* is different from its neighbours in all directions, then this min value will be large.
- If corner strength at a pixel is locally maximal, then this pixel is a corner.

Moravec Corner Detector

- Weakness: If an edge is not aligned with the directions we are looking in, then this edge will be falsely detected as a corner.



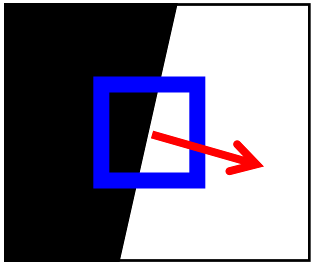
Directions that we look in



Patch is NOT similar to patches in all directions that we look in. **So it will be detected as a corner instead of an edge pixel.**

Harris Corner Detector

- Solution:
  - Do not fix the directions that we are looking in.
  - Find the direction in which patches become most dissimilar.
    - That is, the **direction that maximises the SSD from current patch**.
- This is the idea behind the Harris corner detector.



Taylor's Approximation

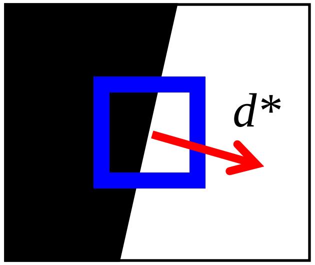
- Functions can be approximated around a point *a* using Taylor's approximation
  f(a+x) = $f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f^{(3)}(a)}{3!}(x-a)^3+\cdots.$

- A 2nd order approximation for 2D functions is
  f(a+x,b+y) ≈

$$f(a,b)+(x-a)\,f_x(a,b)+(y-b)\,f_y(a,b)$$
$$+\frac{1}{2!}\left[(x-a)^2\,f_{xx}(a,b)+2(x-a)(y-b)\,f_{xy}(a,b)+(y-b)^2\,f_{yy}(a,b)\right]$$

Harris Corner Detector

For any patch p, its SSD from a patch separated by a **unit** displacement vector $d = (x,y)$ can be written as



$$S(x,y) = \sum_{u=u_{\min}}^{u_{\max}} \sum_{v=v_{\min}}^{v_{\max}} \left(I(u+x, v+y) - I(u,v) \right)^2$$

Using Taylor's approximation $I(u+x, v+y) \approx I(u,v) + xI_x + yI_y$

$$S(x,y) \approx \sum_{u=u_{\min}}^{u_{\max}} \sum_{v=v_{\min}}^{v_{\max}} \left(xI_x + yI_y \right)^2$$

H.W.: Prove that these two expressions are equivalent.

$$S(x,y) \approx \sum_{u=u_{\min}}^{u_{\max}} \sum_{v=v_{\min}}^{v_{\max}} \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}_{(u,v)} \begin{bmatrix} x \\ y \end{bmatrix} \approx \begin{bmatrix} x & y \end{bmatrix} \left(\sum_{u=u_{\min}}^{u_{\max}} \sum_{v=v_{\min}}^{v_{\max}} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}_{(u,v)} \right) \begin{bmatrix} x \\ y \end{bmatrix}$$

$$S(d) \approx d^T A d$$

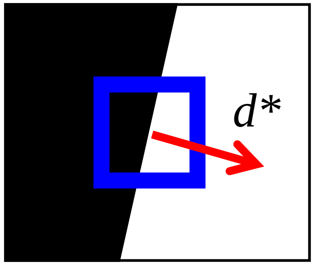
This is the SSD in direction $d = (x,y)$.

We want to find the direction d^* that maximises it.

Harris Corner Detector

Normally a **weighted** SSD is computed

$$S(x,y)=\sum_{u=u_{\min}}^{u_{\max}}\sum_{v=v_{\min}}^{v_{\max}} w(u,v)\big(I(u+x,v+y)-I(u,v)\big)^2$$



Using Taylor's approximation

$$S(x,y)\approx\sum_{u=u_{\min}}^{u_{\max}}\sum_{v=v_{\min}}^{v_{\max}} w(u,v)\begin{bmatrix}x & y\end{bmatrix}\begin{bmatrix}I_x^2 & I_xI_y\\ I_xI_y & I_y^2\end{bmatrix}_{(u,v)}\begin{bmatrix}x\\ y\end{bmatrix}$$

$$S(x,y)\approx\begin{bmatrix}x & y\end{bmatrix}\underbrace{\left(\sum_{u=u_{\min}}^{u_{\max}}\sum_{v=v_{\min}}^{v_{\max}} w(u,v)\begin{bmatrix}I_x^2 & I_xI_y\\ I_xI_y & I_y^2\end{bmatrix}_{(u,v)}\right)}_{A}\begin{bmatrix}x\\ y\end{bmatrix}=d^TAd$$

A is a weighted summation. Can be replaced by (Gaussian) convolution.

$$A=K_\rho*\begin{bmatrix}I_x^2 & I_{xy}\\ I_{xy} & I_y^2\end{bmatrix}=\begin{bmatrix}K_\rho*I_x^2 & K_\rho*(I_xI_y)\\ K_\rho*(I_xI_y) & K_\rho*I_y^2\end{bmatrix}$$

A is the so-called **structure tensor**.

Harris Corner Detector

<u>The Structure Tensor **A**</u>

- A is symmetric ($\mathbf{A}=\mathbf{A}^T$)
- A is positive definite ($\mathbf{v}^T\mathbf{A}\mathbf{v}>0$ for all $\mathbf{v}\neq\mathbf{0}$)
- Eigenvectors of **A** can be made orthonormal.
- Many uses in Computer Vision and Image Processing.

Harris Corner Detector

- $S(\mathbf{d})=\mathbf{d}^T\mathbf{A}\mathbf{d}$ is the SSD in direction **d**.
- We want to find the unit direction vector **d*** that maximises it.
- This is a constrained maximisation problem due to the unit vector constraint on **d**.
- Using the method of Lagrange mulitpliers (cf. Lecture 2)

$$L(d,\lambda)= \mathbf{d}^T\mathbf{A}\mathbf{d}+ \lambda(1-\mathbf{d}^T\mathbf{d})$$

$$0 \equiv \nabla_{\mathbf{d}} L = 2\mathbf{A}\mathbf{d}-2\lambda\mathbf{d}$$ **(H.W. Verify these derivatives?)**

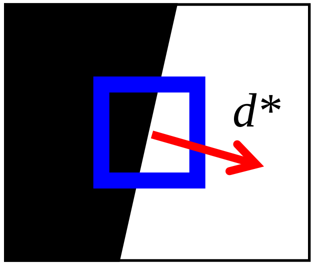
$$\Rightarrow \mathbf{A}\mathbf{d} = \lambda\mathbf{d}$$

$\Rightarrow$ **d*** is an eigenvector of **A**.

- But which eigenvector?

Harris Corner Detector

- Direction **d*** is an eigenvector of **A**. But which one?
- Since **Ad***= λ**d***, multiplying both sides by **d***^{T} we get **d***^{T}**Ad***= $\lambda \Rightarrow \lambda$ is the eigenvalue of **A** corresponding to eigenvector **d***.
- Therefore, for maximising **d**^{T}**Ad**, we must choose **d*** as the eigenvector corresponding to the largest eigenvalue λ_1.



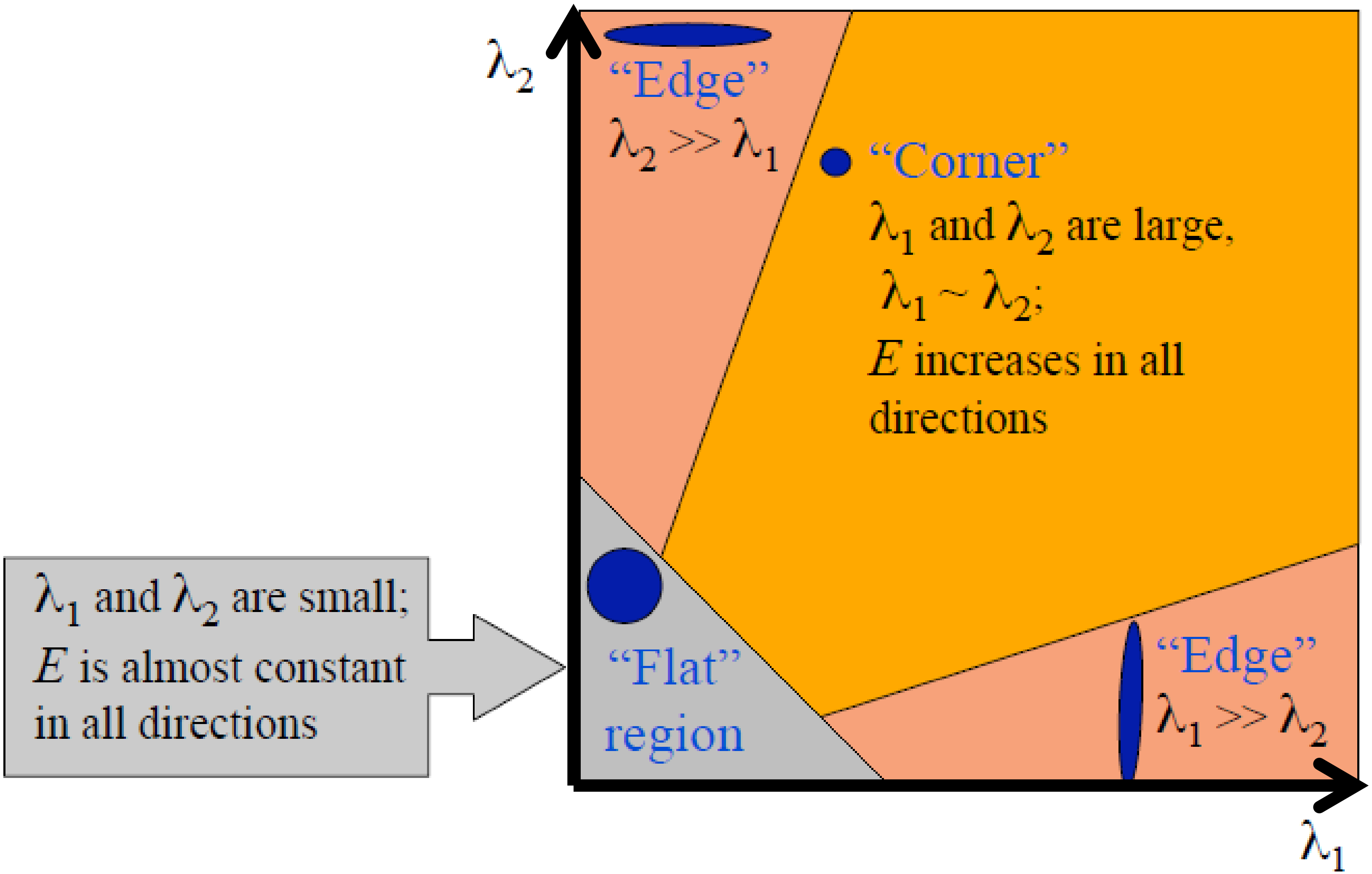
Harris Corner Detector

- The other eigenvector is
  - orthogonal to **d***and
  - represents the direction of least weighted SSD from patch *p*.
- The other (smaller) eigenvalue λ_2 represents the weighted SSD in that direction.
- If $\lambda_2 > T$, patch *p* is a corner patch.

Harris Corner Detector

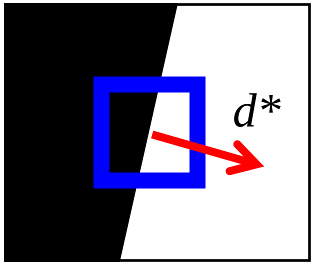
- **If the smaller eigenvalue is large, then we have a non-edge/corner.**
- $\lambda_{large} \approx \lambda_{small} \approx 0 \Rightarrow$ flat region
- $\lambda_{large} >> \lambda_{small} \approx 0 \Rightarrow$ edge
- λ_{large} and $\lambda_{small} >> 0 \Rightarrow$ corner
- $(\lambda_{large} - \lambda_{small})^2$ is a measure of anisotropy

Harris Corner Detector

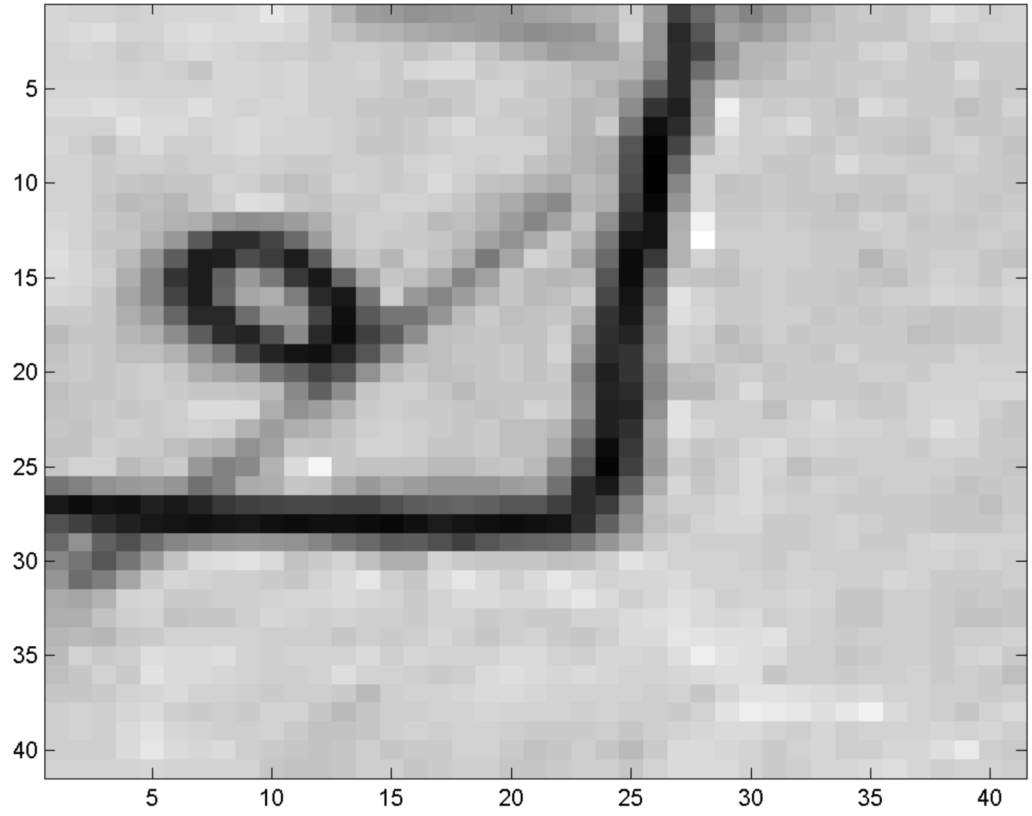


Harris Corner Detector

- What is the difference between $\mathbf{d}^*$ and gradient direction ∇u?
  - Gradient ∇u is for a pixel.
  - $\mathbf{d}^*$ is for a patch.

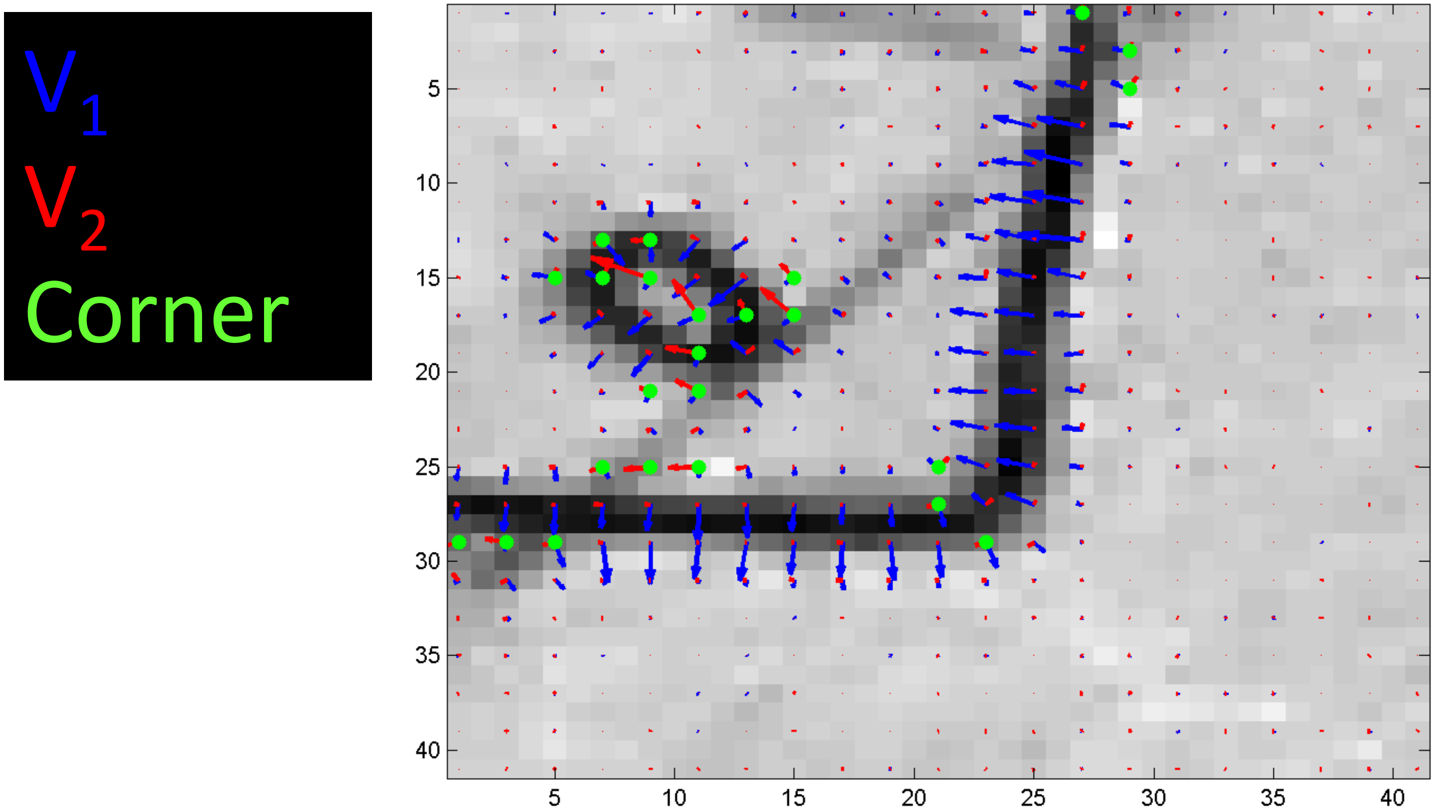


Harris Corner Detector



Author: Nazar Khan (2014)

Harris Corner Detector



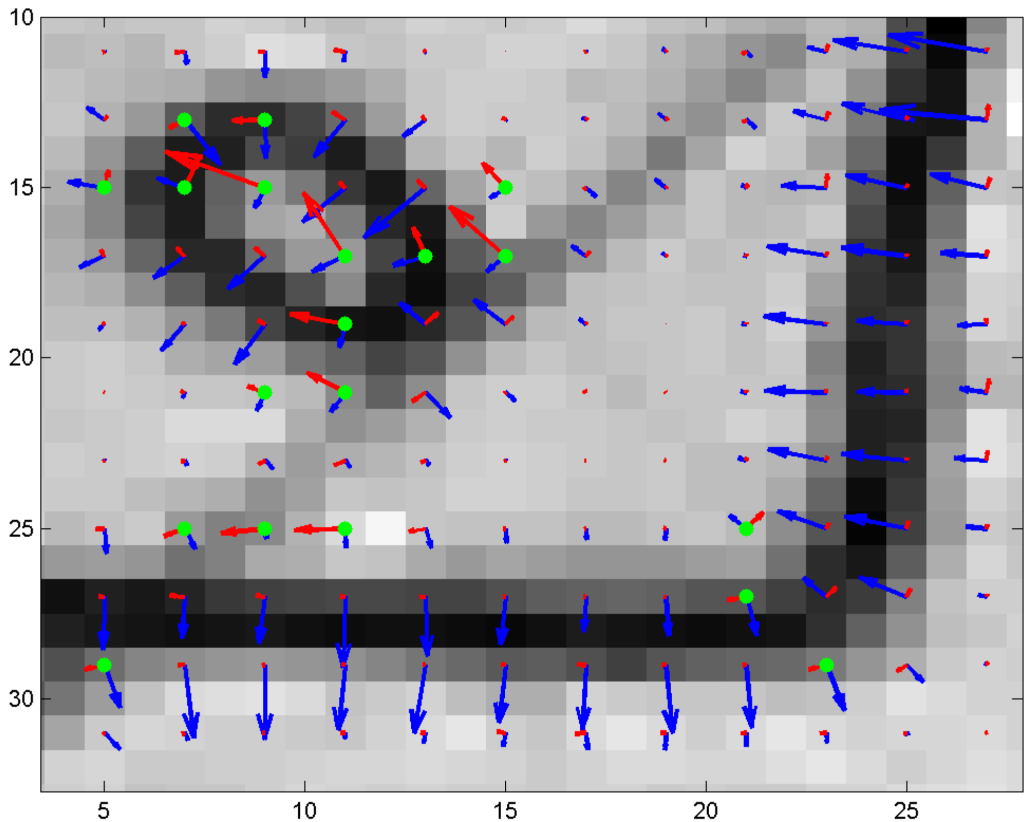
Author: Nazar Khan (2014)

Harris Corner Detector

V_1

V_2

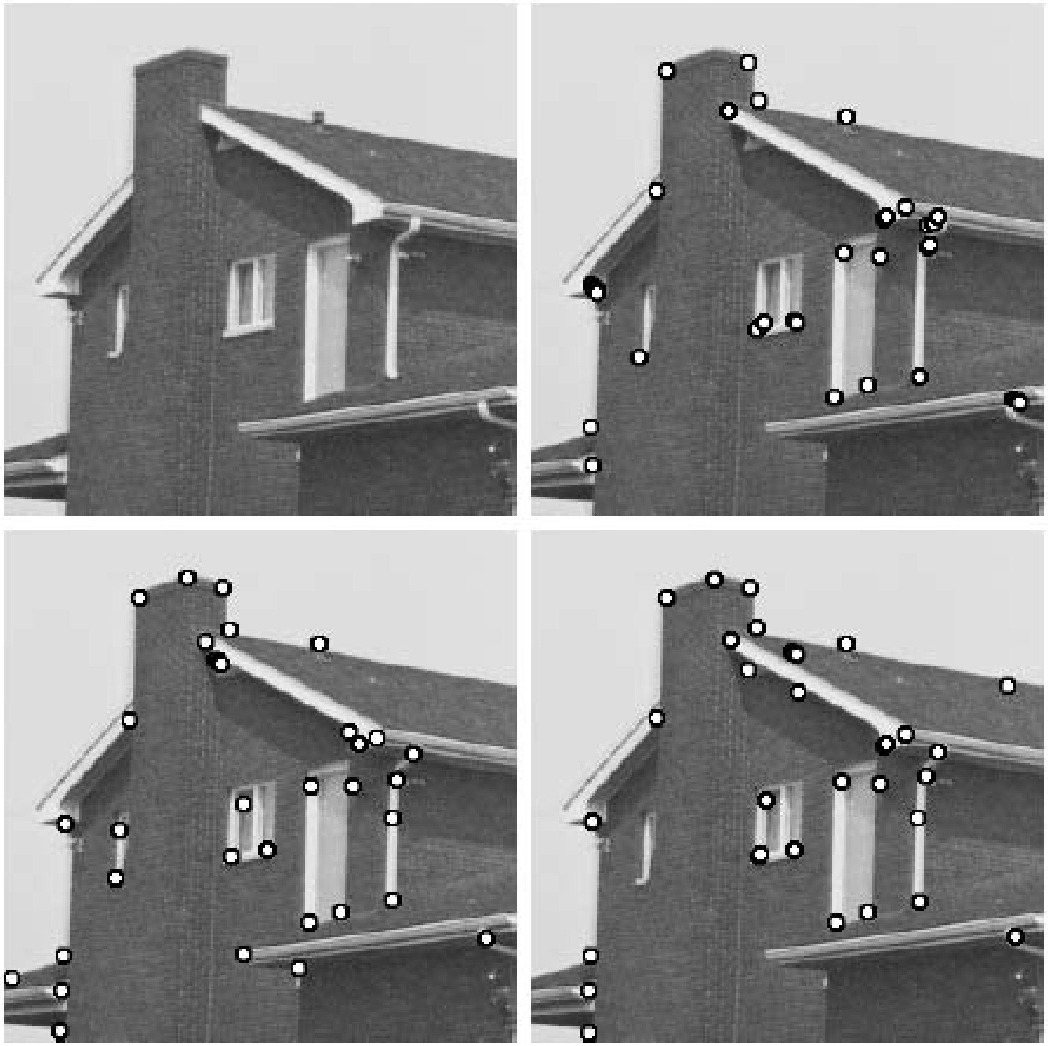
Corner



Author: Nazar Khan (2014)

Corner Detection

- **H.W: Write a detailed derivation/explanation of the Harris corner detector starting from the Moravec corner detector.**
- **H.W: Implement a simple Harris corner detector using the rule $\lambda_{small} > T$ and test it on some sample real-world image.**
  - **Experiment with other corner measures:**
    - **Haris & Stephens**
    - **Shi-Tomasi**
    - **Nobel**



Comparison of structure tensor based corner detectors by choosing the 34 most significant corners for every method ($\sigma = 2$, $\rho = 4$). **Top left:** Original image, 256×256 pixels. **Top right:** Tomasi/Kanade ($T = 18.2$). **Bottom left:** Rohr ($T = 100$) **Bottom right:** Förstner ($T = 72$). Author: J. Weickert (2006).