CS-465 Computer Vision

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11. Camera Geometry

Camera

- A camera projects $3D$ world points to $2D$ *pixel* coordinates.
- In projective space, it is a mapping from $\mathbb{P}^3$ to $\mathbb{P}^2$.
- The whole process of going from $3D$ world coordinates X to $2D$ image pixel coordinates $\mathbf{x}$ can be encoded in a 3×4 camera projection matrix P

  $$\mathbf{x} = PX$$

  where $\mathbf{x}$ is the $2D$ pixel location of the $3D$ world point X when projected by camera P.

Pinhole Camera

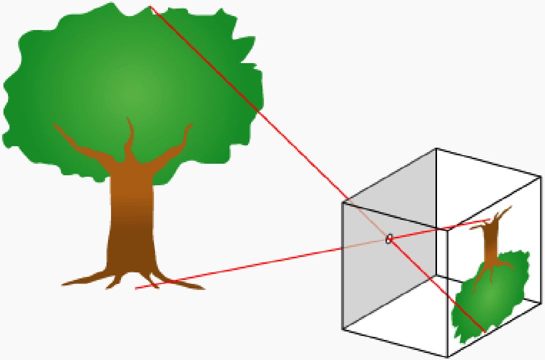
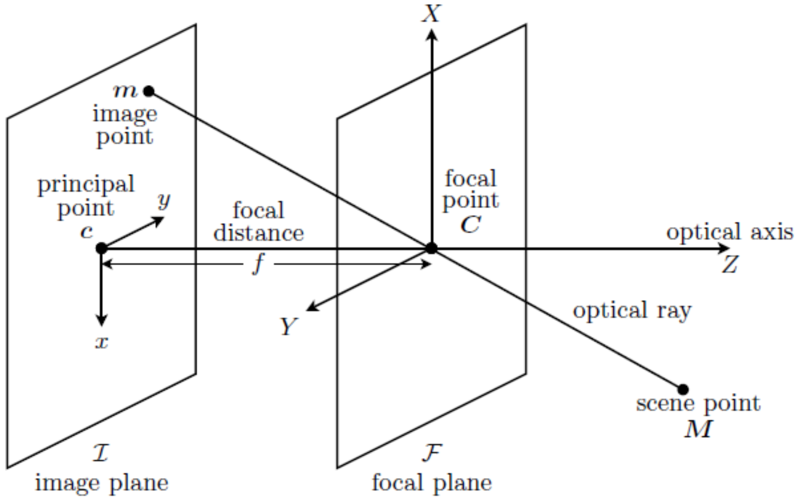


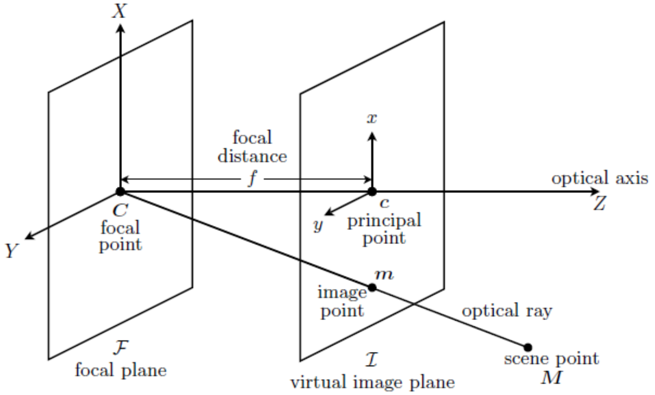
Figure: Pinhole camera with light passing through tiny aperture. Source: Wikipedia

Pinhole Camera Model



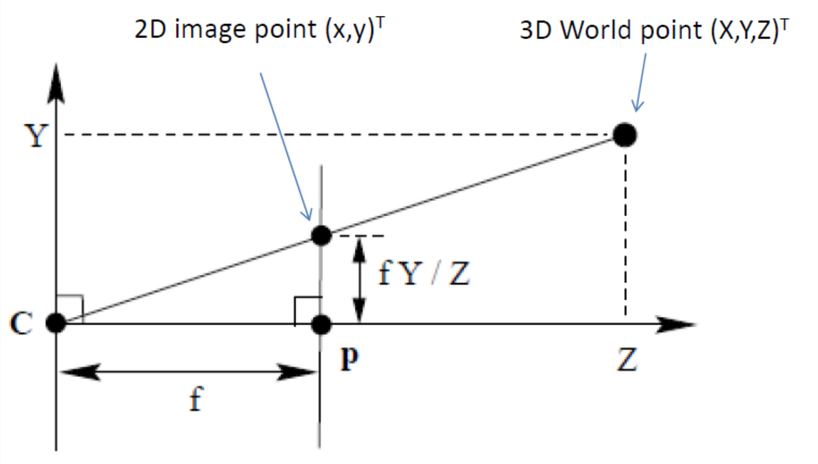
Pinhole camera model. Author: M. Mainberger (2010).

Virtual Image Plane



Pinhole camera model with virtual image plane in front of the focal plane. Author: M. Mainberger (2010).

Camera Projection Equations



$$x = \frac{fX}{Z} \text{ and } y = \frac{fY}{Z}$$

World Coordinates to Camera Coordinates

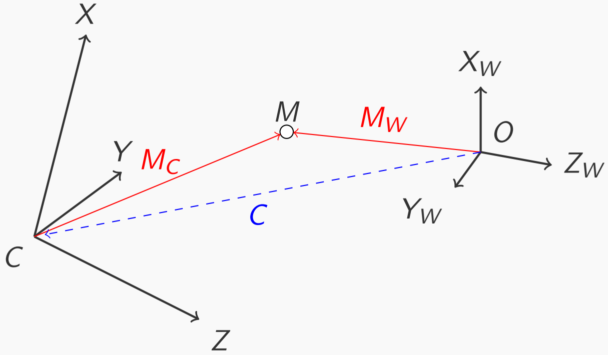
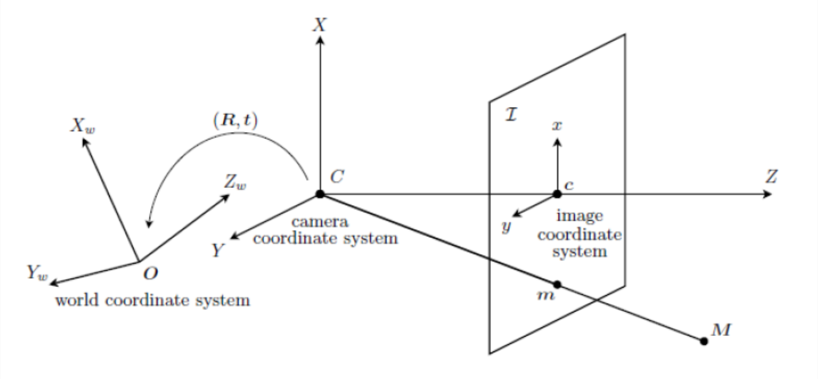
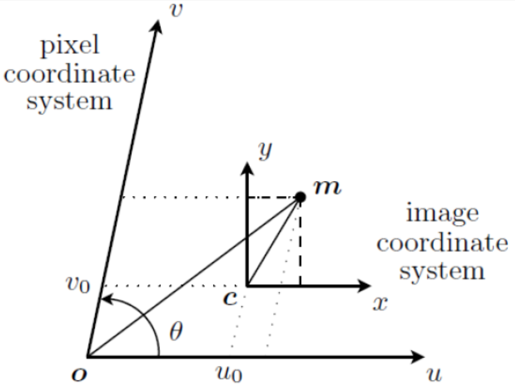


Figure: Any 3D location M has different representations in different coordinate systems. The camera center C itself is a 3D location represented in a world coordinate system. Author: N. Khan (2018)

Extrinsic


X
(R, t)
X_w
$\mathcal{I}$
x
Z_w
C
c
Z
camera
coordinate system
image
coordinate
system
y
Y
Y_w
O
world coordinate system
m
M

Intrinsic



The intrinsic camera parameters describe the transition from the ideal image coordinates $(x, y)^\top$ to the real (pixel) coordinates $(u, v)^\top$. Author: M. Mainberger (2010).

Camera Matrix

- A 3D point in homogeneous world coordinates $(X_w, Y_w, Z_w, 1)^T$ is mapped to a 2D image point with homogeneous pixel coordinates $(u, v, w)^T$ as

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \underbrace{\begin{pmatrix} h_u & -h_u \cot\theta & u_0 \\ 0 & h_v / \sin\theta & v_0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{intrinsic}} \underbrace{\begin{pmatrix} f_x & 0 & 0 & 0 \\ 0 & f_y & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\text{projection}} \underbrace{\begin{pmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\text{extrinsic}} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{pmatrix}}_{\text{full projection matrix}} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix}$$

- 12 parameters in total but with 1 free scaling parameter. So 11 degrees of freedom: 6 extrinsic plus 5 intrinsic.

Anatomy of P

To be completed . . .