CS-565 Computer Vision

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12. Camera Calibration

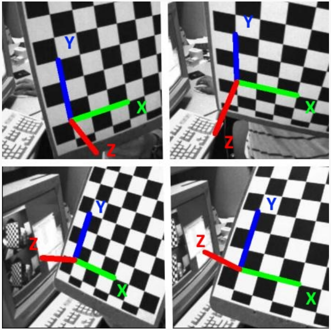
Factorization of the Camera Matrix

- Let K be the 3×3 product of the intrinsic and projection matrices after ignoring the last column of zeros.
- Extrinsic matrix performs a 3×3 rotation R followed by 3×1 translation $\mathbf{t}$.
- We can represent these operations as $[R|\mathbf{t}]$. So $[R|\mathbf{t}]M = RM + \mathbf{t}$.
- Then $P = K[R|\mathbf{t}]$.
- Camera center C is the null-vector of P.

$$PC = 0 \implies K[R|\mathbf{t}]C = 0 \implies KRC + K\mathbf{t} = 0 \implies C = -R^T\mathbf{t}$$

Camera Calibration Using 2D Checkerboard

- Camera calibration is the process of finding the 12 values of P.
- We can use a checkerboard of known size and structure.
- Fix one corner as origin of world coordinate system.
- Then all points on the checkerboard lie in the XY-plane ($Z = 0$).
- Z-axis is orthogonal to the checkerboard.



Camera Calibration Using 2D Checkerboard

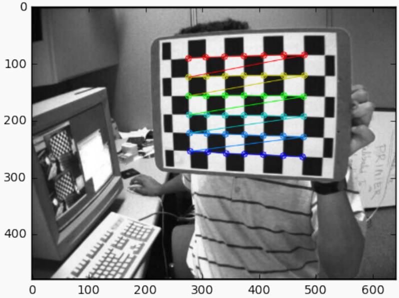
- For any point $\tilde{\mathbf{M}}$ on the checkerboard, its image $\tilde{\mathbf{m}}$ is obtained as

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} K & \mathbf{0}_{3\times 1} \end{pmatrix} \begin{pmatrix} R & \mathbf{t} \\ \mathbf{0}_{1\times 3} & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z = 0 \\ 1 \end{pmatrix} = \underbrace{K \begin{pmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t} \end{pmatrix}}_{\text{Homography } H} \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

- That is $\tilde{\mathbf{m}}_i = H\tilde{\mathbf{M}}_i$.
- Homography for checkerboard image j can be computed via DLT using at least 4 correspondences $\tilde{\mathbf{m}}_i \longleftrightarrow \tilde{\mathbf{M}}_i$.

Camera Calibration Using 2D Checkerboard

- The $\mathbf{m}_i$ can be found using corner detection.



- The corresponding $\mathbf{M}_i$ can be computed since the size and structure of the checkerboard are known (squares of measured dimensions).

Camera Calibration Using 2D Checkerboard

- For the estimated homography $H = \begin{bmatrix}\mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3\end{bmatrix}$, we can write

$$\begin{bmatrix}\mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3\end{bmatrix} = K\begin{bmatrix}\mathbf{r}_1 & \mathbf{r}_2 & \mathbf{t}\end{bmatrix}$$

- So $\mathbf{r}_i \equiv K^{-1}\mathbf{h}_i$.
- Since $\mathbf{r}_1$ and $\mathbf{r}_2$ are columns of a rotation matrix

$$\begin{aligned}
\mathbf{r}_1^T\mathbf{r}_2 = 0 &\implies \mathbf{h}_1^T K^{-T}K^{-1}\mathbf{h}_2 = 0 \\
\mathbf{r}_1^T\mathbf{r}_1 = 1 &\implies \mathbf{h}_1^T K^{-T}K^{-1}\mathbf{h}_1 = 1 \\
\mathbf{r}_2^T\mathbf{r}_2 = 1 &\implies \mathbf{h}_2^T K^{-T}K^{-1}\mathbf{h}_2 = 1
\end{aligned}$$

- The term $K^{-T}K^{-1}$ is a symmetric and positive definite matrix that we denote by B.

$$B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{pmatrix}$$

Camera Calibration Using 2D Checkerboard

- So for each homography (*i.e.*, checkerboard image), we have two constraints

$$\mathbf{h}_1^T B \mathbf{h}_2 = 0$$
$$\mathbf{h}_1^T B \mathbf{h}_1 - \mathbf{h}_2^T B \mathbf{h}_2 = 0$$

- For all images, these constraints can be written as a linear system $V\mathbf{b} = 0$ from which $\mathbf{b} = \begin{pmatrix} b_{11} & b_{12} & b_{22} & b_{13} & b_{23} & b_{33} \end{pmatrix}^T$ can be found via SVD.
- Matrix K can be recovered through the Cholsky decomposition of B.

$$\text{chol}(B) = AA^T$$

which means that $K = A^{-T}$.

Camera Calibration Using 2D Checkerboard

- Once K is found, the extrinsic parameters can be found easily.

$$\begin{aligned} \mathbf{r}_1 &\equiv K^{-1}\mathbf{h}_1 \\ \mathbf{r}_2 &\equiv K^{-1}\mathbf{h}_2 \\ \mathbf{t} &\equiv K^{-1}\mathbf{h}_3 \end{aligned}$$

  and after normalising $\mathbf{r}_1$ and $\mathbf{r}_2$, we can find the third column of R as $\mathbf{r}_3 \equiv \mathbf{r}_1 \times \mathbf{r}_2$.
- Notice that K is computed from B which is computed using the constraint $\mathbf{h}_1^T B \mathbf{h}_1 = \mathbf{h}_2^T B \mathbf{h}_2$.
- This means that $\mathbf{r}_1$ and $\mathbf{r}_2$ will have the same magnitude m which represents the homogeneous scale of matrix K.

Camera Calibration Using 2D Checkerboard

- So the actual rotation and translation vectors are

$$\mathbf{r}_1 \leftarrow \frac{1}{m}\mathbf{r}_1$$
$$\mathbf{r}_2 \leftarrow \frac{1}{m}\mathbf{r}_2$$
$$\mathbf{r}_3 \leftarrow \mathbf{r}_1 \times \mathbf{r}_2$$
$$\mathbf{t} \leftarrow \frac{1}{m}\mathbf{t}$$