MA-120 Probability and Statistics

Nazar Khan
PUCIT

Lecture 6: Probability

Probability

- One of the most important branches of Mathematics.
- Many problems can be reformulated in terms of a probabilistic framework.
- Can be rather unintuitive.
- Has its own language and terminology.
- **Motivation: Chance error!**
  - Real world always has chance errors. How do we get the best predictions in presence of such uncertainty.

Outline

- Quantifying Uncertainty
- Terminology
  - Experiment
  - Outcome
  - Sample space
  - Event
- Set Theory
- Mutual Exclusion
- Axioms of Probability

Quantifying Probabilities

- Experiments can be of 2 types
  - Deterministic (we can accurately predict outcome)
  - Random (we cannot accurately predict outcome)
- Prediction is the hallmark of science.
- It is what separates humans from animals.
- Probability theory plays a fundamental role in prediction when the experiment is random.

Three key concepts

1. Sample Space
2. Events
3. Probabilities of events

The next few slides will introduce some terminology that is **crucial** for understanding Probability. So pay attention!

Terminology

- **Outcome** of the random experiment is denoted by the symbol ω.
- **Sample space** – the set of all possible outcomes is denoted by the set S.
  - For coin toss, $S=\{H,T\}$.
  - For a roll of the die, $S=\{1,2,3,4,5,6\}$.
  - Select a number between 0 and 1, $S=[0,1]$.

Terminology

- **Event** – a statement concerning the elements of the sample space
  - Even number on the die, S=[1,2,3,4,5,6] and A=[2,4,6].
- An event is always an element of the sample space. (WHY?)
- Outcomes ω that agree with the statement form the event.
- **Probability** deals with assigning numbers to events.

Terminology

- Set Theory
  - Union
  - Intersection
  - Complement

Terminology

Random Experiment

- Anything that produces an uncertain output.
- Tossing a coin, rolling a die, voting in elections, etc.

Outcome

- What an experiment produces.
- Coin landing heads, die giving a 6, election resulting in People’s Party winning.

Terminology

Events

- The class E of all events that we are interested in is also called a sigma field. It obeys the following axioms
  1. S is always considered an event,
  2. If A is an event then A^c must also be considered as an event,
  3. A countable union of events must also be an event. That is, if $A_1, A_2, \cdots$ are all events then $A_1 \cup A_2 \cup \cdots$ must also be an event.

Terminology

When we perform a random experiment.

- S = sample space
- E = events in the sample space
- P = real-valued probability function for events E.
  - $P(E) \epsilon [0,1]$

Probability Space: The collection (S,E,P).

Axioms of Probability

The probability function P obeys the following axioms:

1. $0 \le P(A) \le 1$ for any event A,
2. $P(S) = 1$ and
3. If A_1, A_2, ... are mutually exclusive events then $P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots$

Properties of P

If A,B are events, then

1. $P(\emptyset) = 0$, (impossibility property)
2. $P(A^c) = 1 - P(A)$, (complement property)
3. $P(A^c \cap B) = P(B) - P(A \cap B)$, (more general complement property)
4. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, (union property)
5. If $A \subseteq B$, then $P(A) \leq P(B)$, (monotonicity property)