MA-120 Probability and Statistics

Nazar Khan
PUCIT
Lecture 8: Statistical Independence

Independence

- Terminology:
  - **Joint probability P(A,B)**: probability of some events occurring together.
  - **Marginal probability P(A)**: probability of a single event.

Independence

Example
Let two dice (red and green) be rolled so that all the 36 possible outcomes are equally likely. Let A be the event that the red die lands 4, and let B be the event that the green die lands 4.

- S=?
- Is S simple?
- $|S|=?$
- $|A|=?$
- $|B|=?$
- $|A \cap B|=?$

Independence

$P(A) = 6/36$
$P(B) = 6/36$
$P(A)P(B) = 6/36*6/36 = 1/36$
$P(A \cap B) = 1/36 = P(A)P(B)$.

- Clearly, A and B are independent events. What we get on the red die has no influence on what we get on the green die.
- For independent events the joint probability equals the product of marginal probabilities.
- Conversely, when $P(A \cap B) = P(A)P(B)$, events A and B must be independent.

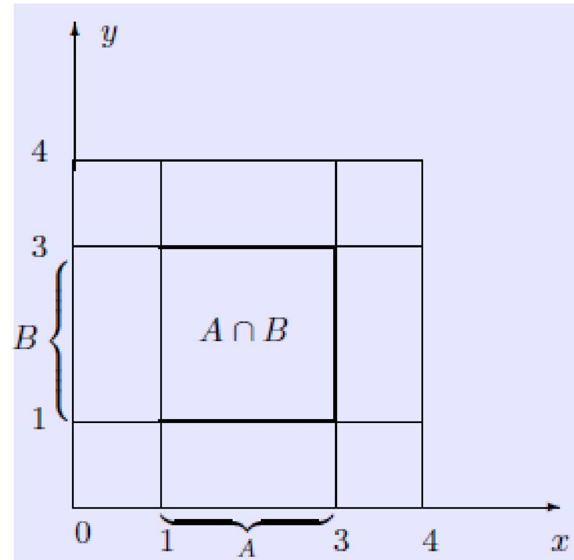
Independence

- There are 2 ways to look at the independence concept.
  - Intuitive
  - Probabilistic
- Intuitively, two events are independent if they do not influence or block the occurrence of each other.
- Probabilistically, two events are independent if joint probability equals the product of marginal probabilities.

Independence

Example

Let a point be selected at random from a square, [0, 4] × [0, 4].
Let A be the event that the selected point lies in the rectangle [1, 3] × [0, 4].
Let B be the event that the selected point lies in the rectangle [0, 4] × [1, 3].
Are A and B independent?



P(A)=? P(B)=? P(A∩B)=?

Independence

Example

The sample space corresponding to the gender of 3 children of a family is as follows.

S = {BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG}.

Assume S is a simple sample space. Let A be the event that the family has both boys and girls, and let B be the event that the family has at most one girl. Are A and B independent events?

Independence

A = family has both boys and girls = {BBG, BGB, GBB, BGG, GBG, GGB},

B = family has at most one girl = {BBB, BBG, BGB, GBB}.

Therefore, $P(A) = 6/8$, $P(B) = 4/8$ and $P(A \cap B) = 3/8 = P(A)P(B)$.

H.W: Check the independence when the family has only 2 children.

Independence

Example

Let a card be drawn from a well shuffled deck of 52 cards. Let A be the event that the drawn card is an ace. Let B be the event that the drawn card is a club. Show that A and B are independent events.

Example

We select a card at random from the standard deck of 52 cards, let A be the event that a face-card is drawn (i.e., a Jack, or Queen or King is drawn). Let B be the event that the drawn card is a club. Show that A and B are independent events.

Mutual Independence

Pairwise independence

If $A_1, A_2, \cdots$ are events, then this sequence is called pairwise independent if every pair of two events is independent.

Mutual independence

If $A_1, A_2, \cdots$ are events, then this sequence is called mutually independent if every finite subset of events is independent. That is

$$P\left(\bigcap_{j \in J} A_j\right) = \prod_{j \in J} A_j$$

for every finite subset J of {1,2,3,…}.