CS-568 Deep Learning

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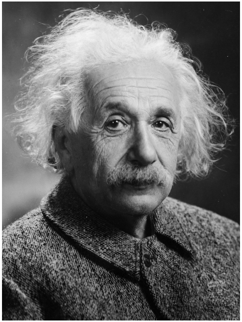
PUCIT

Recurrent Neural Networks



Everything should be made as simple as possible, but no simpler.

Albert Einstein

Understanding Recurrent Neural Networks requires some effort and a correct perspective. Do not expect them to be as simple as linear regression.

Static vs. Dynamic Inputs

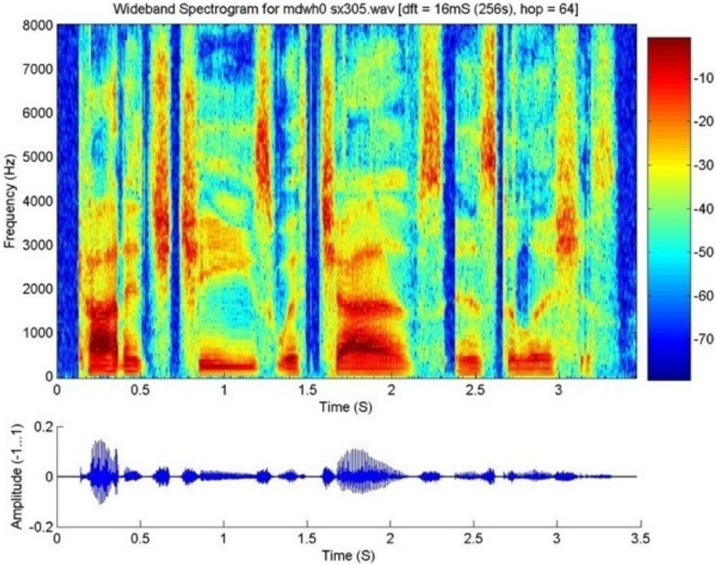
- *Static* signals, such as an image, do not change over time.
  - Ordered with respect to space.
  - Output depends on current input.
- *Dynamic* signals, such as text, audio, video or stock price change over time.
  - Ordered with respect to time.
  - Output depends on current input as well as past (or even future) inputs.
  - Also called *temporal*, *sequential* or *time-series* data.

Context in Text

The Taj _____ was commissioned by Shah Jahan in 1631, to be built in the memory of ___ wife Mumtaz Mahal, who died on 17 June that *year, giving birth to* their *14th child, Gauhara Begum.* Construction *started in 1632, and the mausoleum was completed ___ 1643.*

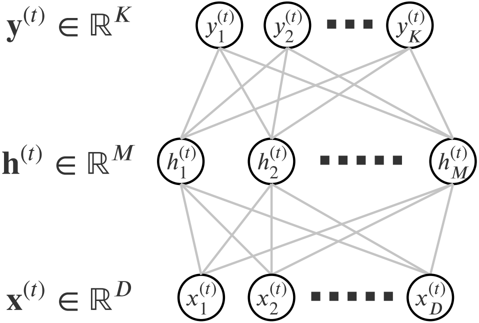
Context in Video

Context in Audio



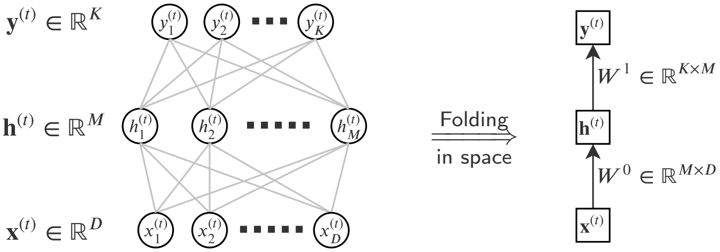
Time-series Data

- A *single* input will be a *series of vectors* $\mathbf{x}^1, \mathbf{x}^2, \ldots, \mathbf{x}^T$.



Input component at time t forward propagated through a network.

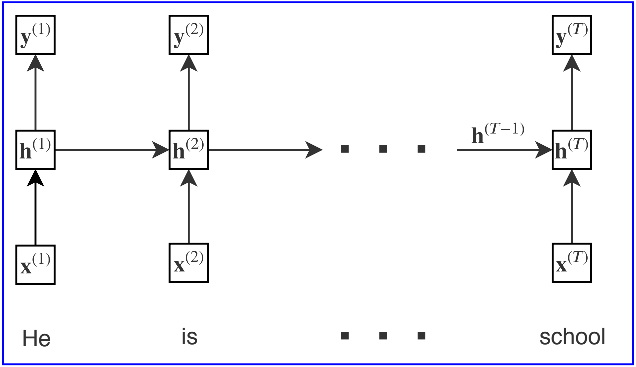
Representational Shortcut 1 – Space Folding



Each box represents a layer of neurons.

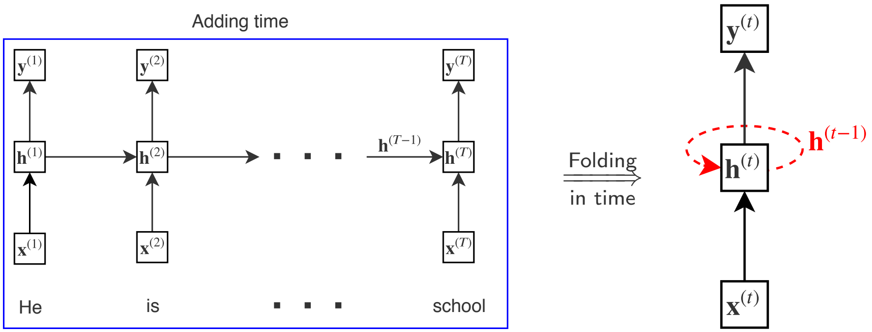
Recurrent Neural Networks

Adding time



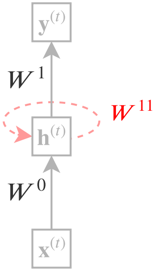
- A recurrent neural network (RNN) makes hidden state at time t directly dependent on the hidden state at time $t-1$ and therefore indirectly *on all previous times*.
- Output $\mathbf{y}_t$ depends on all that the network has already seen so far.

Representational Shortcut 2 – Time Folding



Recurrent Neural Networks

3 sets of weights

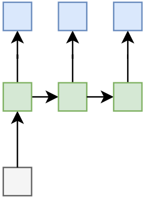


$$\mathbf{y}^{(t)} = f(\overbrace{W^1\mathbf{h}^{(t)} + \mathbf{b}_1}^{\mathbf{a}^{1(t)}})$$

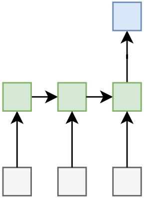
$$\mathbf{h}^{(t)} = \tanh(\underbrace{W^0\mathbf{x}^{(t)} + W^{11}\mathbf{h}^{(t-1)} + \mathbf{b}_0}_{\mathbf{a}^{0(t)}})$$

Sequence Mappings

One-to-many



Many-to-one



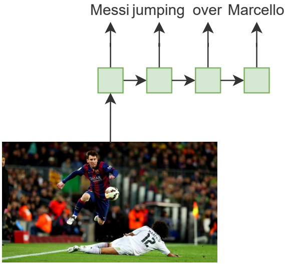
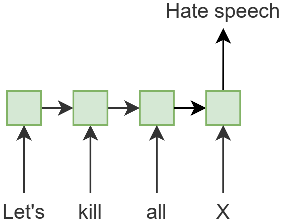
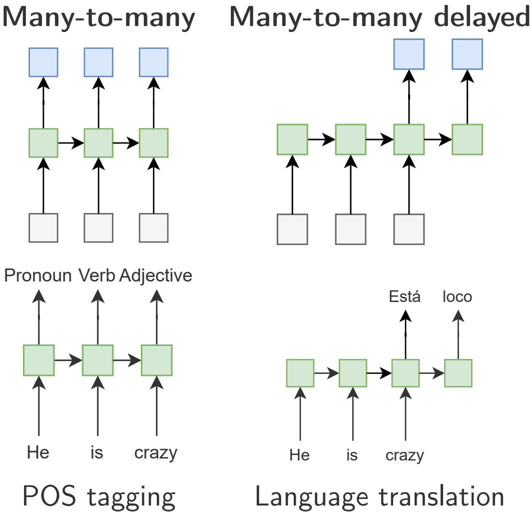


Image caption generation



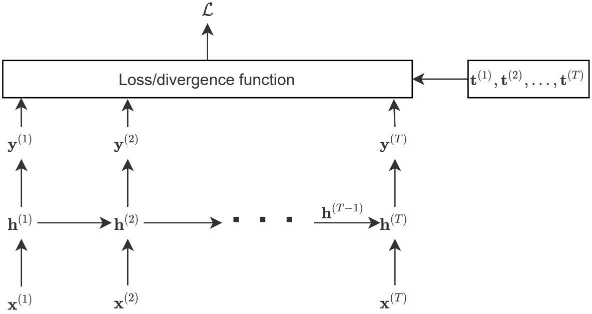
Sentiment classification

Sequence Mappings



Loss Functions for Sequences

- For recurrent nets, loss is between *series* of output and target vectors. That is $\mathcal{L}(\{\mathbf{y}^{(1)}, \ldots, \mathbf{y}^{(T)}\}, \{\mathbf{t}^{(1)}, \ldots, \mathbf{t}^{(T)}\})$.

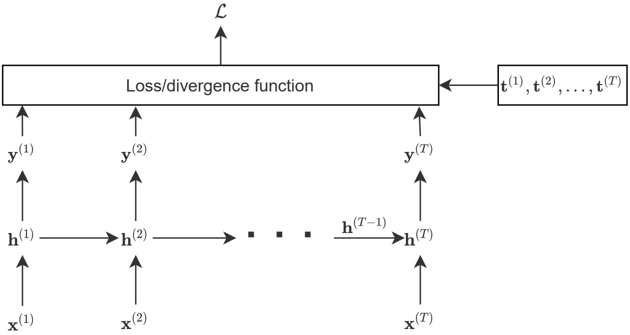


Forward propagation in an RNN unfolded in time.

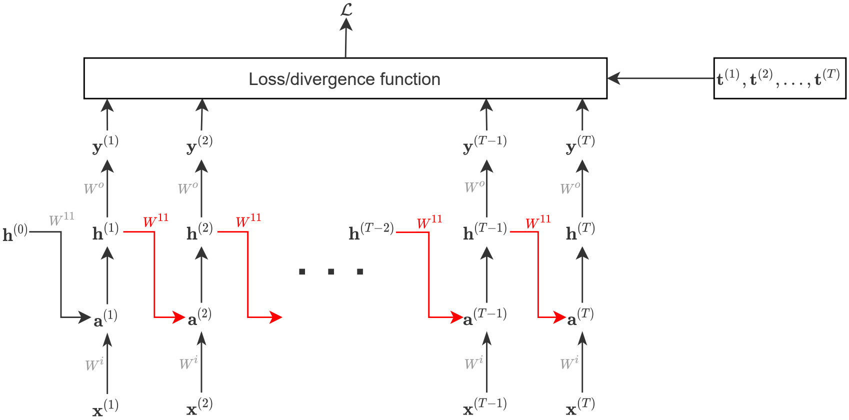
- Notice that loss $\mathcal{L}$ can be computed only after $\mathbf{y}^{(T)}$ has been computed.

Loss Functions for Sequences

- Loss is *not necessarily* decomposable.
- In the following, we will assume decomposable loss $\mathcal{L} = \sum_{t=1}^{T} \mathcal{L}(\mathbf{y}^{(t)}, \mathbf{t}^{(t)})$.
- In both cases, as long as $\frac{\partial L}{\partial \mathbf{y}^{(t)}}$ has been computed, backpropagation can proceed.

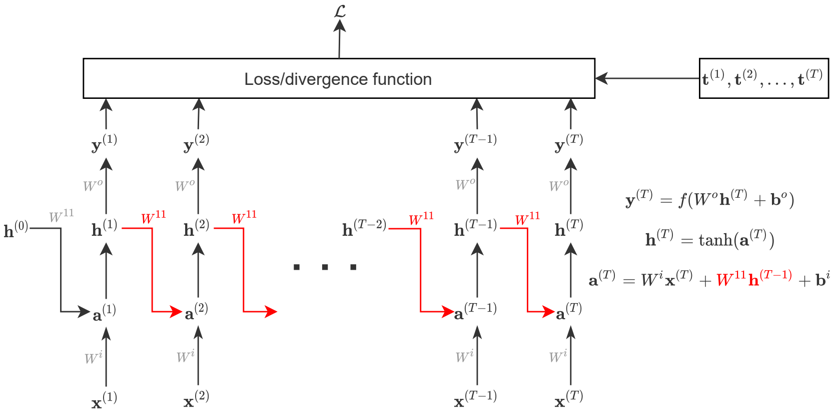


Forward Propagation Through Time



Forward propagation in an RNN unfolded in time. Recurrence between hidden states through pre-activation $\mathbf{a}^{(t)}$ is shown in red.

Forward Propagation Through Time



$$\mathbf{y}^{(T)} = f(W^o\mathbf{h}^{(T)} + \mathbf{b}^o)$$

$$\mathbf{h}^{(T)} = \tanh(\mathbf{a}^{(T)})$$

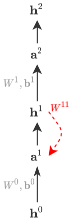
$$\mathbf{a}^{(T)} = W^i\mathbf{x}^{(T)} + W^{11}\mathbf{h}^{(T-1)} + \mathbf{b}^i$$

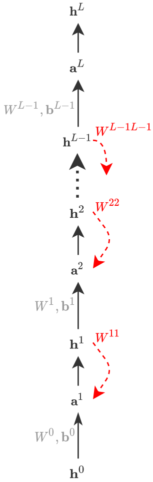
Forward propagation in an RNN unfolded in time. Recurrence between hidden states through pre-activation $\mathbf{a}^{(t)}$ is shown in red.

Notational Clarity

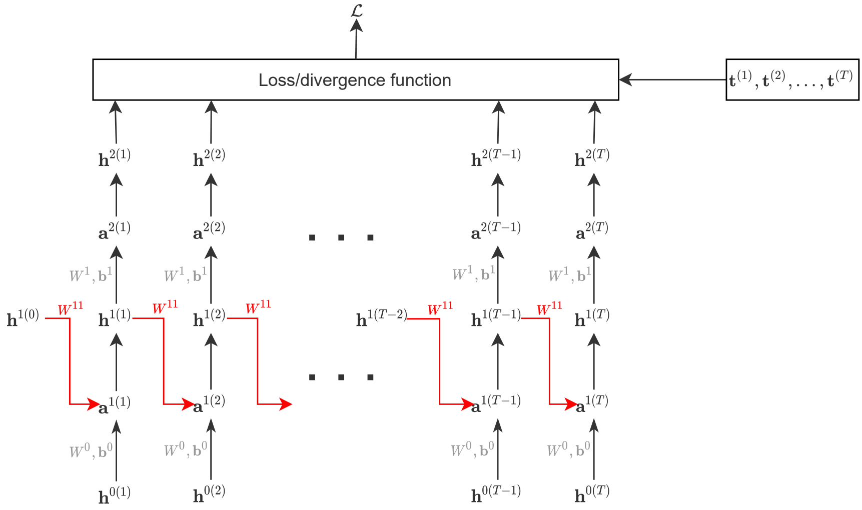
- At layer l, we will denote the pre-activation by $\mathbf{a}^l$ and activation by $\mathbf{h}^l$.
- So output layer $\mathbf{y}$ will be denoted by $\mathbf{h}^L$ in an L-layer network.
- Input will be denoted by $\mathbf{h}^0$.
- So forward propagation entails $\mathbf{h}^0 \rightarrow \underbrace{\mathbf{a}^1 \rightarrow \mathbf{h}^1}_{\text{layer }1} \cdots \rightarrow \underbrace{\mathbf{a}^{L-1} \rightarrow \mathbf{h}^{L-1}}_{\text{layer }L-1} \rightarrow \underbrace{\mathbf{a}^L \rightarrow \mathbf{h}^L}_{\text{layer }L}$.
- For 2 layer network

$$
\begin{aligned}
\mathbf{h}^{2,T} &= f(\mathbf{a}^{2,T}) \\
\mathbf{a}^{2,T} &= W^1\mathbf{h}^{1,T} + \mathbf{b}^1 \\
\mathbf{h}^{1,T} &= \tanh(\mathbf{a}^{1,T}) \\
\mathbf{a}^{1,T} &= W^0\mathbf{h}^{0,T} + {\color{red}W^{11}\mathbf{h}^{1,T-1}} + \mathbf{b}^0
\end{aligned}
$$

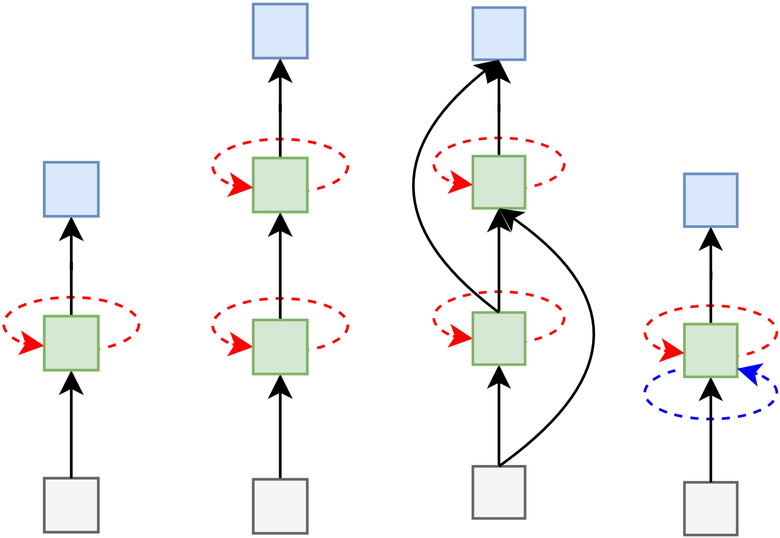




Notational Clarity



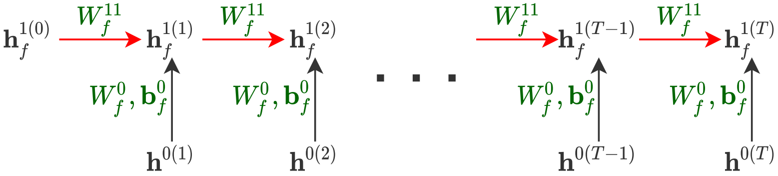
RNN Variations



1 hidden state 2 hidden states Skip connections Bidirectional

Bidirectional RNN

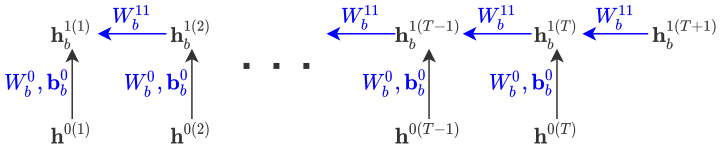
Step 1: Forward propagation into the future



$$\boxed{fprop\left(\underbrace{\mathbf{h}^0(1), \mathbf{h}^0(2), \dots, \mathbf{h}^0(T)}_{\text{input sequence}}; \underbrace{\mathbf{h}_f^1(0)}_{\text{init}}, \underbrace{W_f^0, \mathbf{b}_f^0, W_f^{11}, W_f^1, \mathbf{b}_f^1}_{\text{parameters}}\right)}$$

Bidirectional RNN

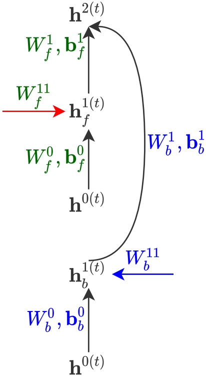
Step 2: Forward propagation into the past



$$\boxed{fprop\left(\underbrace{\mathbf{h}^0(T), \mathbf{h}^0(T-1), \ldots, \mathbf{h}^0(1)}_{\text{input sequence}}; \underbrace{\mathbf{h}_b^1(T+1)}_{\text{init}}, \underbrace{W_b^0, \mathbf{b}_b^0, W_b^{11}, W_b^1, \mathbf{b}_b^1}_{\text{parameters}}\right)}$$

Bidirectional RNN

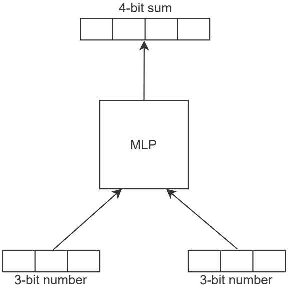
Step 3: Fusion of forward and backward hidden states



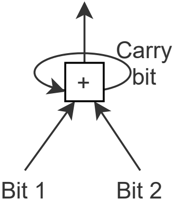
$$\mathbf{h}^{2(t)} = \tanh(\mathbf{a}^{2(t)})$$
$$\mathbf{a}^{2(t)} = W_f^1 \mathbf{h}_f^{1(t)} + \mathbf{b}_f^1 + W_b^1 \mathbf{h}_b^{1(t)} + \mathbf{b}_b^1$$

Benefit of Recurrent Architectures

n-bit addition

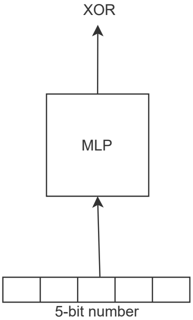


- Only for n-bit numbers
- Training set exponential in n
- Training errors

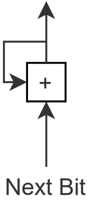


- Iterative application to numbers of arbitrary size
- Exact answers

Benefit of Recurrent Architectures
n-bit XOR



- Only for n-bit numbers
- Training set exponential in n
- Training errors



- Iterative application to numbers of arbitrary size
- Exact answer
- Will be 1 for odd number of ones in input.

Stability issues

- Even a 1-hidden layer RNN is a very deep network.
- Viewed in time, an RNN is as deep as the number of time steps.
- Suffers from vanishing gradients.
- Also suffers from *exploding gradients*.

Exploding Gradients

- Consider input $\mathbf{x}^{(1)}$ at time 1 and *assume linear* hidden layer.
- At time t, the RNN carries a term of the form

$$W^{11} \dots W^{11} W^{0} \mathbf{x}^{(1)} = \left(W^{11}\right)^{t-1} W^{0} \mathbf{x}^{(1)}$$

  which is an M-dimensional vector.
- Magnitude of this vector depends on largest eigenvalue $\lambda_{\max}$ of W^{11}.
  - $\lambda_{\max} > 1 \implies$ magnitude of $\left(W^{11}\right)^{t-1} W^{0} \mathbf{x}^{(1)}$ keeps increasing.
  - $\lambda_{\max} < 1 \implies$ magnitude of $\left(W^{11}\right)^{t-1} W^{0} \mathbf{x}^{(1)}$ keeps decreasing.

Exploding Gradients

- Even during forward propagation, depending on the largest eigenvalue of the recurrent weight matrix W^{11}, input at time t
  - is either forgotten very soon,
  - or explodes to very large values.
- Similar case for backpropagation.
- Notice that this has nothing to do with the choice of activation function.
- *Information will explode or vanish through time*.
- Similar behaviour for non-linear neurons.
- So, in practice, RNNs do not have long-term memory. Solution: LSTM.