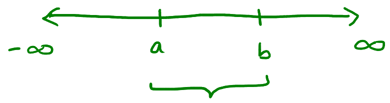
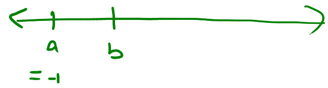
① Uniform Random Variable



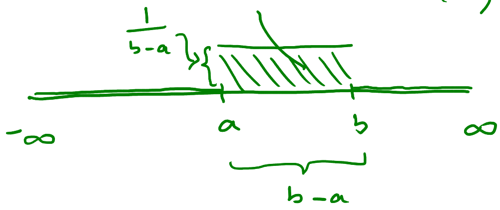


X = any number between a and b

$$f(x) = \begin{cases} \frac{1}{b-a} & , a < x < b \\ 0 & , \text{otherwise} \end{cases}$$

Area $= (b-a)\frac{1}{(b-a)}$

$= 1$



① Non-negative ✓

② Total area under density function is 1. ✓

$$\text{Total area} = \int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{a} f(x)dx + \int_{a}^{b} f(x)\,dx + \int_{b}^{\infty} f(x)\,dx$$

$$= \int_{-\infty}^{a} 0\,dx + \int_{a}^{b} \frac{1}{b-a}\,dx + \int_{b}^{\infty} 0\,dx$$

$$= 0 + \frac{1}{b-a}\int_{a}^{b} 1.dx + 0$$

$$= \frac{1}{b-a}\left(x\Big|_a^b \right) = \frac{1}{b-a}(b-a) = \frac{b-a}{b-a} = 1$$ Total

Discrete Uniform R.V

Fair-coin H , T
$\frac{1}{2}$ $\frac{1}{2}$ ← f(x)

Fair-die 1, 2, 3, 4, 5, 6
$\frac{1}{6}$ $\frac{1}{6}$ . . . . . $\frac{1}{6}$ ← f(x)

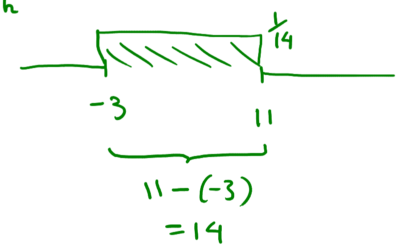
$Y \sim \text{Uniform}(-3, 11)$ (a = −3, b = 11)

$$f(y) = \begin{cases} \frac{1}{11-(-3)} & , -3 < y < 11 \\ 0 & , \text{otherwise} \end{cases}$$

$A = w \times h$

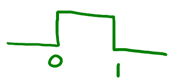
$1 = (b-a) \times h$

$h = \frac{1}{b-a}$



$X \sim U(0,1)$

$[0,1]$



Random-number generate

→ Pseudo-random number

→ $U(0,1)$

$X \sim U(0,5)$

$U(0,1) \times 5$

$X \sim U(-1,5)$

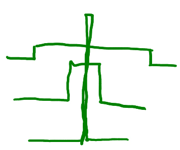
$\underbrace{U(0,1) \times 6}_{[0,6]} - 1$

$[-1, 5]$

$U(a,a)$ ← Is it random?

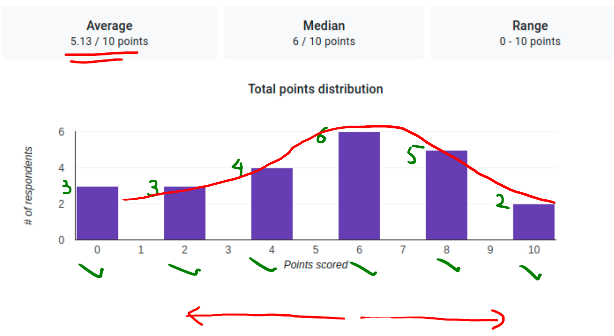
$$f(x) = \begin{cases} \frac{1}{a-a} & , a \le x \le a \\ 0 & , \text{otherwise} \end{cases}$$

$\frac{1}{0} = \infty$



$X =$ quiz marks

$\Delta = \{0, 2, 4, 6, 8, 10\}$



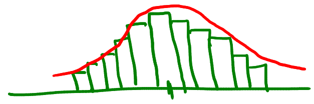
زیادہ تر values اپنی average کے پاس ہوتی ہیں اور جیسے جیسے average سے دور جائیں، chances کم ہو جاتے ہیں۔

Normally random variables are distributed this way.

250 ml

250 ml $\pm \epsilon$ ← random

1000 bottles



250

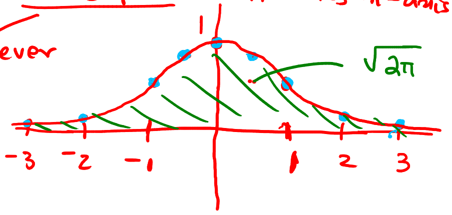
$$f(x) = e^{-\frac{x^2}{2}}$$

Assymptotically approaches x-axis

$1^2 > 1$

$-1^2 > 1$

will never touch.



$\sqrt{2\pi}$

-3 -2 -1 1 2 3

$f(x)$ is non-negative

$$e^{-\frac{(2000)^2}{2}} = \frac{1}{e^{\text{very big number}}} = \frac{1}{\text{even bigger number}} \to 0$$

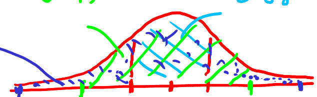
$$\boxed{f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}}$$

$\sqrt{2\pi}$

① Non-neg.
② Area 1

} Density function ← Standard normal

0.997 0.95 0.68



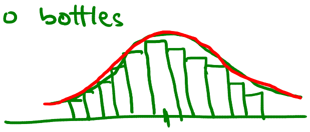
-3 -2 -1 0 1 2 3

$\boxed{\text{68 - 95 - 99 rule}}$

Machine 1

1000 bottles

250 ml $\pm \epsilon_1$

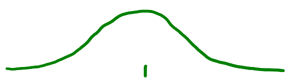


250

σ^2 large

x: ←250→ $\qquad$ $x - 250$: ←0→

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-250)^2}{2}} \leftarrow$$



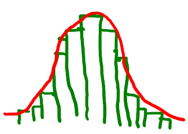
250

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2}} \leftarrow$$

μ determines where you place the bell. ← the peak of the bell.

Machine 2

250 ml $\pm \epsilon_2$



250

σ^2 small

$$\boxed{\mathcal{N}(x; \mu, \sigma^2) \quad f(x) = \frac{1}{\sqrt{2\pi\underbrace{\sigma^2}_{\text{new}}}} e^{-\frac{(x-\mu)^2}{2\underbrace{\sigma^2}_{\text{new}}}}}$$

← Normal density function

$X \sim \mathcal{N}(\mu, \sigma^2)$

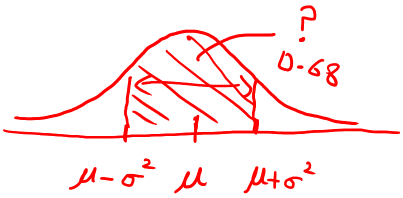
σ^2 determines the width of the bell.

$$\mathcal{N}(x;\underbrace{0,1}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \leftarrow$$ Standard normal density

$X \sim \mathcal{N}(x;\ \mu, \sigma^2)$

$Y = X - \mu \sim \mathcal{N}(y;\ 0, \sigma^2)$

$Z = \frac{X-\mu}{\sigma} \sim \underbrace{\mathcal{N}(z;\ 0, 1)}_{\text{Standard normal}}$



σ^2 = variance

σ = standard deviation

μ

$z = \frac{x-\mu}{\sigma} \leftarrow$ z-transformation

standardization

z-score

z-score tells us how many standard deviations away from its mean was x.

$X < \begin{matrix}\mu = 10 \\ \sigma = 2\end{matrix}$

$\mu + 2\sigma = 14$

$10 + 2(2)$

$\mathcal{N}(x;\mu,\sigma^2)$

Standardize $\downarrow$

$\mathcal{N}(0,1)$

Quiz 1

$x_1 = 14$ $\mu_1 = 10$

$\sigma_1 = 2$

$x = 14$

$\frac{14-\mu}{\sigma} = \frac{14-10}{2} = \frac{4}{2} = 2$

$z_1 = \frac{x_1 - \mu_1}{\sigma_1} = 2$

Student was 2 SDs above the mean.

Quiz 2

$x_2 = 12$ $\mu_2 = 10$

$\sigma_2 = 1$

$z_2 = \frac{x_2 - \mu_2}{\sigma_2} = \frac{12-10}{1} = \frac{2}{1} = 2$