Spread of a random variable

Finance — volatility

Statistics — standard deviation

$$E(X^2) = \sum_{x\in\Delta} x^2 P(X=x) \leftarrow \text{2nd-moment of } X.$$

$$E(X^k) = \sum_{x\in\Delta} x^k P(X=x) \leftarrow k\text{-th moment of } X.$$

$$\underset{\substack{\uparrow \\ \sigma^2,\ \sigma_X^2}}{\text{Variance}}(X) = E(\underbrace{(X-E(X))^2}_{f(X)})$$

$$= E\big((X-E(X))(X-E(X))\big)$$

$$= E\big(X^2 - 2XE(X) + (E(X))^2\big)$$

$$= E(X^2) - 2E(X)\,E(X) + E\big((E(X))^2\big)$$

$$= E(X^2) - 2(E(X))^2 + (E(X))^2$$

$$= \underbrace{E(X^2)}_{\text{2nd-moment}} - \underbrace{(E(X))^2}_{\text{square of 1st moment}}$$

$E(aX+b) = aE(X)+b$

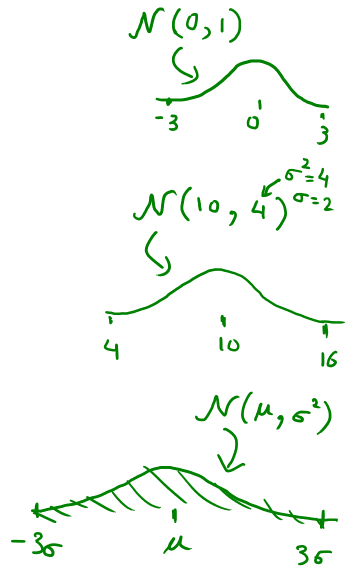
$E(aX+bY) = aE(X)+bE(Y)$

$\leftarrow E(X^2) \geq (E(X))^2$

$\Rightarrow Var(X) \geq 0$

$$\underset{\substack{\uparrow \\ \sigma,\ \sigma_X}}{\text{Standard Deviation}} = \sqrt{Var(X)}$$

Almost all probability lies between $\mu \pm 3\sigma$.



Properties of Var(X)

1. $Var(X) \geq 0$
2. If $var(X)=0$, then X is not random.
3. $Var(aX) = a^2 Var(X)$

   └ $Std.Dev(aX) = a\,Std.Dev(X)$
4. $Var(X \pm c) = Var(X)$

If $X \sim Bin(n,p)$

Then $E(X) = np$

$Var(X) = np(1-p)$

$\big(\underbrace{E(X^2)} - \underbrace{(E(X))^2}_{np}$

Same reasoning.

μ $\quad$ $\mu+10$

Gauss, Euler, Newton, ⋮ — Contributions to many many different fields.

Queen of Densities

Gaussian Density

0	1	2	.	.	.	n

$Bin(n,p)$ $\to$ μ, σ^2 $\to$ $N(np, np(1-p))$

Approximate

$N(\mu, \sigma^2)$

Normal approximation

μ, σ^2 — Bin — Poisson .. — Discrete — μ, σ^2

Continuous — μ, σ^2