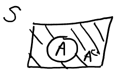
Some consequences of the 3 axioms

① $P(\emptyset)=0$ $\quad \emptyset=\{\}$

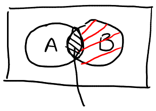
$\left[\ \underbrace{P(\emptyset)+P(S)}_{LHS} = P(\emptyset\cup S) = \underbrace{P(S)}_{R.H.S} \Rightarrow P(\emptyset) = P(S)-P(S) = 1-1 = 0 \right.$



② $P(A^c)=1-P(A)$

$\left[\ P(A^c)+P(A) \underset{\uparrow \text{ axiom 3}}{=} P(\underbrace{A^c\cup A}_{S}) = P(S) \underset{\uparrow \text{ axiom 2}}{=} 1 \Rightarrow P(A^c)=1-P(A) \right.$

③ $P(A^c\cap B) = P(B)-P(A\cap B)$



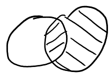
$A^c\cap B$ is set B excluding set A.

$P(\underbrace{\text{red}}_{A^c\cap B})+P(\underbrace{\text{black}}_{A\cap B}) = P(\underbrace{\text{red} \cup \text{black}}_{B})$

$P(A^c\cap B) = P(B)-P(A\cap B)$

$P(B\setminus A)$

$\cancel{P(B)\setminus P(A)}$



~~B − A~~ = ?

$A\cup B$

$A\cap B$

$B\setminus A$

$A\setminus B \leftarrow -$

$\downarrow$

\ set-difference

3 − 2 = 1

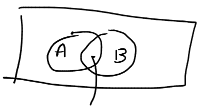
12 − 20 = −8

{2,4,6} − {integers} = ?

↑

?

④ $P(A\cup B) = P(A)+P(B)-P(A\cap B)$



$A\cup B$ contains all elements in A and B.

A ______________ in A including those that are in $A\cap B$

B ______________ in B ______________ $A\cap B$

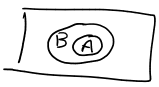
$P(A)+P(B)$ counts $A\cap B$ twice.

$P(A\cup B) = P(A)+P(B)-P(A\cap B)$

⑤ If $A\subseteq B$, then $P(A)\leq P(B)$

$A\subset B$

↑



[If A occurs then B also occurs.

But if B occurs then A might or might not occurs.

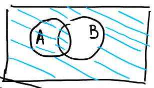
Therefore, likelihood of occurance of A can never be more than ______________ B.

That is, $P(A)\leq P(B)$

Examples

$2+3+4$
$2+4+3$

① If $P(A\cup B) = 0.6$ and $P(A\cup B^c) = 0.8$, find $P(A)$?



$P(A\cup B) = P(A)+P(B)-P(A\cap B)$

لگے رہیں

$$\underline{(A\cup B)\cup(A\cup B^c)} = A\cup B\cup A\cup B^c = \underbrace{A\cup A}_{A}\cup B\cup B^c$$
$$= A\cup \underbrace{B\cup B^c}_{S} = \underbrace{A\cup S}_{S} = S$$

$$P(\underbrace{(A\cup B)\cup(A\cup B^c)}_{S}) = P(A\cup B) + P(A\cup B^c) - P(\underbrace{(A\cup B)\cap(A\cup B^c)}_{A})$$

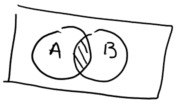
$A \leftarrow$ convince yourself.

$$\underbrace{P(S)}_{1} = \underbrace{P(A\cup B)}_{0.6} + \underbrace{P(A\cup B^c)}_{0.8} - \underbrace{P(A)}_{?}$$

$$P(A) = 0.6+0.8-1 = \underline{\underline{0.4}}$$

② Explain why $P(A\cap B) \le \min(P(A), P(B)) \le \max(P(A), P(B)) \le P(A\cup B)$?

obvious



$P(A\cap B) = P(A) \leftarrow \min(P(A), P(B))$

$A \subseteq A\cup B \Rightarrow P(A) \le P(A\cup B)$
$B \subseteq A\cup B \Rightarrow P(B) \le P(A\cup B)$

③ $P(B) = 0.3$
$P(A\cup B^c) = 0.6$
$P(A) = ?$

$P(B^c) = 0.7$

If $P(B^c) = 0.7$ then $P(A\cup B^c) \ge 0.7$!!!

But question states $P(A\cup B^c) = 0.6$.

Question is invalid

Probability follows logic and common sense. One cannot just assign probabilities based on opinion.

Then how should one assign probabilities? Next few lectures.