Methods for computing probability

Sample Space

Finite

Coin-toss $\{H,T\}$

Die roll $\{1,2,3,4,5,6\}$

Integers from a to b $\{a, a+1, a+2, \dots, b-1, b\}$

45 50

Set "size" can be defined via counting of elements.

$$P(A) = \frac{\#\text{ elements in } A}{\#\text{ elements in } S}$$

Infinite

Countably infinite

Any integer $\{\text{All integers}\}$

Positive integer $\{1, 2, \dots, \infty\}$

$\to$ Pos. Int. $\subset$ All integers

Smaller of larger infinities

Integers > 100

$\{101, 102, \dots, \infty\}$

$\downarrow \quad \downarrow$

$1 \quad 2 \quad \dots$

Set "size" can be defined via "sums"?

Uncountably infinite

(bounded)

Infinitely large

$\downarrow$

$[0,1]$

Calculus?

Real $(-\infty, \infty)$

Real numbers from 0 to 1

a b

$0 \quad 0.1 \quad 0.2 \quad \dots \quad 1$

$\downarrow \quad \downarrow \quad \downarrow \quad \quad \downarrow$

$1 \quad 2 \quad 3 \quad \dots$

$[0,1]$

$0 \quad 0.01 \quad 0.02 \quad \dots$

$\downarrow \quad \downarrow \quad \downarrow$

$1 \quad 2 \quad 3$

$0 \quad 0.001 \quad 0.002$

$\downarrow \quad \downarrow \quad \downarrow$

$1 \quad 2 \quad 3$

Set "size" can be defined via measurement

- length
- area
- volume

$$P(A) = \frac{\text{"size" of } A}{\text{"size" of } S}$$

where definition of "size" depends on the kind of sample space.

- "size" can be "counted" or "measured".

- Counting can be difficult, time consuming and error-prone.
- But there are some tricks and short-cuts to counting.

Q. A fair die is rolled 3 times. What is the probability that all faces will be different?

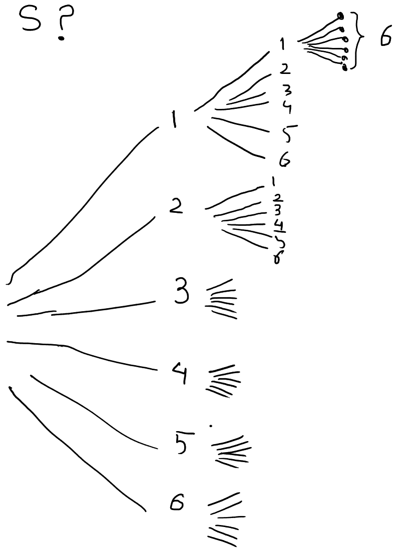
- what is the sample space S?

$$S = \{ \underbrace{(1,2,3)}_{\text{triples}}, \underbrace{(2,4,6)}_{\text{triple}}, \dots \}$$

- Is the sample space simple?

  Yes, bcz. the die is fair

- Size of S?

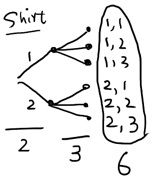


6   6x6   6x6x6

This is the multiplication principle.

When a task is completed in stages (say 2 stages) and the first stage can be completed in m ways and the second stage can be completed in n ways, then the whole task can be completed in mn ways.

2 shirts
3 jeans

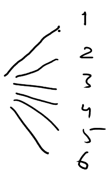


Multiplication principle

$$|S| = 6 \times 6 \times 6 = 6^3 = 216$$

- $|A| = ?$

  If I had to "force" event A, then in how many ways could I have complete this task?



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6 options   5 options   4 options

$$|A| = 6 \times 5 \times 4 = 120$$

- $P(A) = \frac{|A|}{|S|} = \frac{120}{216} = 0.5555$

Q. There are n people in a room. You have to select 3 people.
One of them will be the president.
____________ secretary
____________ treasurer.
How many ways can you make such a selection?

Ans.

Order matters.

President	Secretary
1.	2, 3, …, n — $n-1$
2	
3	
4	
⋮	
n	

① Permutation

Ordered arrangement of distinct objects.

3 objects a, b & c.

$\underline{3}\ \underline{2}\ \underline{1} = 6$

Mult. princ.

$$\boxed{\begin{matrix} a\,b\,c \\ a\,c\,b \\ b\,a\,c \\ b\,c\,a \\ c\,a\,b \\ c\,b\,a \end{matrix}}$$

Enumerated

6 permutations of 3 objects a, b & c.

correspond to the same

② "combination".

order does not matter

$$\underline{n} \times \underline{n-1} \times \underline{n-2} = n(n-1)(n-2)$$

Permutation is just the multiplication principle.

$$P^n_3 = P(n,3) = n(n-1)(n-2) = \frac{n(n-1)(n-2)(n-3)\cdots 1}{(n-3)(n-4)\cdots 1}$$

$$= \frac{n!}{(n-3)!}$$

$$P^n_k = \frac{n!}{(n-k)!} = n(n-1)(n-2)\cdots(n-(k-1))$$

Q. In how many ways can you pick a, b, c from a, b, c when their order of arrangement matters?

$n = 3$

$k = 3$

$$P(3,3) = \frac{3!}{(3-3)!} = \frac{3!}{0!} = \frac{3\cdot 2\cdot 1}{1} = 6$$

Q. If there are 3 people, what is the prob. that no two people share the same day of birth?

$1, \ldots, 365$ — $1, \ldots, 365$

– $S = ?$

$$S = \{ \underset{\text{triple}}{(13, 257, 59)},\ \underset{\text{triple}}{(1, 1, 1)},\ \ldots \}$$

– Is S simple? Yes

– $|S| = 365 \times 365 \times 365 = 365^3$

– $|A| = 365 \times 364 \times 363$

$$P(A) = \frac{365 \times 364 \times 363}{365^3}$$

$= 0.9918$

Next time

"Combinations"