Counting can be used for simple sample spaces.

- Finite
- Equilikely

Multiplication Principle

Permutation — ordered arrangements of distinct objects.

$$P^n_k = \frac{n!}{(n-k)!}$$

a b c
a c b
b a c
b c a
c a b
c b a

} Permutations

Combination

Unordered arrangement of distinct objects.

3 objects
$3 \cdot 2 \cdot 1 = 6$ permuts.
$3! = 6$ (1 combination)

k objects.
$k(k-1)(k-2)\cdot \cdots 1$
$= k!$
(1 combination)

a b c
a c b
b a c
b c a
c a b
c b a

} 6 Permutations correspond to just 1 combination.

$$C^n_k = \frac{P^n_k}{k!}$$

a b c d

In how many ways can you pick 3 objects out of these 4 so that order matters?

$$P^4_3 = \frac{4!}{4-3!} = 24$$

$$C^4_3 = \frac{24}{3!} = 4$$

$3! = 6$ {a b c ⋮} 1 combination

$3! = 6$ {a b d ⋮} 1 combination

$3! = 6$ {a c d ⋮} 1 combination

$3! = 6$ {b c d ⋮} 1 combination

6 {a b d; b a d; d b a; d a b; b d a; a d b}

a b c d

How many ways can you pick 2 objects out of 4 when order matters & when order does not matter?

$$P^4_2 = \frac{4!}{2!} = 12$$

$$C^4_2 = \frac{12}{2!} = 6$$

12 {a b; b a} {a c; c a} {a d; d a} {b c; c b} {b d; d b} {c d; d c}

$\frac{12}{2} = 6$ Combinations

4 H.w

3 objects
2 object
$P^3_2 = \frac{3!}{1!} = 6$

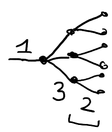
4 distinct objects.

— a $\boxed{b \quad c \quad d}$. You have to pick 3.

- Fix "a" at the 1st position.

How many ways can you pick the remaining 2.

a → $\underset{\uparrow}{1} \quad \underset{\uparrow}{3} \quad \underset{\uparrow}{2} = 6$



- Common sense
- A little of thought
- Multiplication principle

You will solve most of these questions.

you win 100 rupees

a b c
a c b ← 0 rupees

a b c d — a — Without replacement

a b c d — a — with replacement

(a b c)

P(100 rupees) = ?

— b —

a b c
a c b
b c a
b a c
c b a

$= \frac{\text{size}(A)}{\text{Size}(S)}$ = counted / counted

Q. You have 3 apples and 4 oranges in a basket.
The fruits are all mixed up.
You draw 3 items without looking.

① $S = \{\underbrace{---}_{\text{triples}}, \underbrace{---}_{\text{triple}}, ---, \dots\}$

② Is S simple?

Yes, because the items are mixed up.

(i) What is the size of the sample space S?

In how many ways can you draw 3 items from $3+4=7$ items?

– Does ordering of items matter/count or not?

a a a o o o o

a o o

a o o
o a o
o o a

$a_1\ a_2\ a_3\ o_1\ o_2\ o_3\ o_4$

$$|S| = C^7_3 = \frac{P^7_3}{3!} = \frac{7!}{(7-3)!\,3!} = \frac{7\cdot 5\cdot \cancel{6}\cdot \cancel{4!}}{\cancel{4!}\ \cancel{3!}} = 35$$

$$\frac{7\times 6\times 5}{3!} = 35$$

$a_1\ a_2\ o_4$
$a_2\ a_1\ o_4$

(ii) In how many ways can you select 2 apples and 1 orange?

$E = \{2 \text{ apples and } 1 \text{ orange}\}$

$\to a_1\ a_2\ o_4$

$\dfrac{3\times 2\times 1}{3!} = 6$

Stage 1: Pick 2 apples
Stage 2: Pick 1 orange

$a_1\ a_2\ a_3\ o_1\ o_2\ o_3\ o_4$

Stage 1: $C^3_2 = \dfrac{3!}{1!\,2!} = 3$

Stage 2: $C^4_1 = \dfrac{4!}{3!\,1!} = 4$

$$|E| = \underbrace{C^3_2}_{3} \times \underbrace{C^4_1}_{4} = 3\times 4 = 12$$

① $\to a_1\ a_2$ o_1, o_2, o_3, o_4

② $\to a_1\ a_3$ o_1, o_2, o_3, o_4

③ $\to a_2\ a_3$ o_1, o_2, o_3, o_4

$\}\ 12$

(iii) P(2 apples and 1 orange) = P(E) = ?

$$P(E) = \frac{|E|}{|S|} = \frac{12}{35}$$

S was simple