

Simple sample space

Counting methods

Multiplication Principle

Permutation

Combination

$$C_k^n = \frac{P_k^n}{k!}$$

a b c d

a b c
a c b
b a c
b c a
c a b
c b a

3!

a b d

↓

3!

a c d

↓

3!

b c d

↓

3!

24 permutations

4 groups
4 combinations
 $C_3^4 = \frac{4!}{1!3!} = 4$

Q. A box of apples.

G good apples.

B bad apples.

We reach in and draw n apples all at once.

What is the probability that we get k good apples?

Ans. $S = \{ \underbrace{\text{---}}_{n\text{-tuple}}, \underbrace{\text{---}}_{n\text{-tuple}}, \dots \}$

Is S simple?

Yes

$$|S| = C_n^{B+G} = \binom{B+G}{n}$$

→ E = { k good apples and n-k bad apples }

To find |E|, imagine if you were "forced" to form event E, then how many ways could we have?

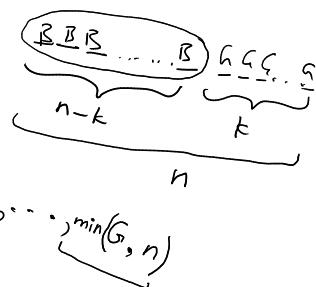
~~$\binom{n}{k}$~~ $|E| = \binom{G}{k} \times \binom{B}{n-k}$

$$P(E) = P(\text{k good apples in n apples}) = \frac{|E|}{|S|} = \frac{\binom{G}{k} \times \binom{B}{n-k}}{\binom{G+B}{n}}, k=0,1,2,\dots,\min(G,n)$$

Hypergeometric model

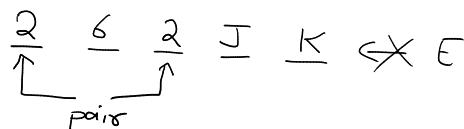
When does the experiment end?

When you get n apples.



Q. Consider a well-shuffled deck of 52 playing cards. I randomly draw 5 cards.

Find the probability of getting 2 pairs.

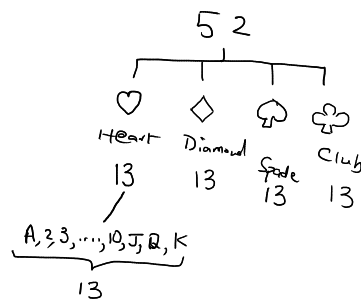


$$P(E) = \frac{|E|}{|S|} = \frac{\binom{13}{2} \binom{4}{2} \binom{4}{2} \binom{44}{1}}{\binom{52}{5}}$$

Stage 1
pick the pair categories

Stage 2
pick 2 cards from each category

Stage 3
pick last card from the remaining categories.



numbers?

$$S = \{ \underbrace{\text{---}}_{5\text{-tuple}}, \underbrace{\text{---}}_{5\text{-tuple}}, \dots \}$$

Is S simple?

Yes

$$|S| = \binom{52}{5}$$

$$|E| = \binom{13}{2} \times \binom{4}{2} \times \binom{4}{2} \times \binom{44}{1}$$

choose 2 categories from 13

choose 2 cards from 4 of 1st category

choose 2 cards from 4 of 2nd category

choose 1 card from 44 cards.

Q. Roll a die 10 times and record their outcomes.

$$S = \{ \underbrace{\quad\quad\quad}_{10\text{-tuple}}, \dots \}$$

S is simple.

$$|S| = 6^{10} = \underbrace{6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6}_{10 \text{ times}}$$

Event E is

6 repeated 2 times	
1 repeated 3 times	
2	5
↑	↑

$$P(E) = ?$$

$$|E| = \binom{10}{2} \times \binom{8}{3} \times \binom{5}{5} \quad \begin{array}{cccccccc} _ & _ & _ & _ & _ & _ & _ & _ \\ 6 & 6 & & & & & 6 & 6 \end{array}$$

$$= \binom{10}{2} \times \binom{10-2}{3} \times \binom{10-2-3}{5} = \frac{10!}{\boxed{8!} \cdot 2!} \times \frac{\boxed{8!}}{\boxed{5!} \cdot 3!} \times \frac{\boxed{5!}}{0! \cdot 5!}$$

$$= \frac{10!}{2! \cdot 3! \cdot 5!}$$

$$\leftarrow \frac{n!}{k_1! \cdot k_2! \cdot k_3! \cdots k_m!} = \binom{n}{k_1, k_2, \dots, k_m}$$

$$P(E) = \frac{10!}{2! \cdot 3! \cdot 5! \cdot 6^{10}}$$

↑
multinomial
coefficient

$$C_k^n = \binom{n}{k}$$

Binomial coefficient