— Simple — Counting

Countably infinite sample space

$S = \{1, 2, 3, \ldots, \infty\}$

$a + ar + ar^2 + ar^3 + \cdots = k < \infty$

$-1 < r < 1$

$S = \{ \underset{\#_1}{\_}, \underset{\#_2}{\_}, \underset{\#_3}{\_}, \ldots \}$

size(S) = sum(#'s)

size(A) = sum(#'s assigned to elements of A)

$|S| = ?$

$$\begin{array}{ccccccc} 1 & 2 & 3 & 4 & \cdots & \infty \\ \downarrow & \downarrow & \downarrow & \downarrow & & \\ 1 & \frac{1}{2} & \frac{1}{4} & \frac{1}{8} & \cdots & \end{array}$$

Trick: Assign sizes.

size(S) $\equiv$ sum of the assigned numbers

$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$

geometric series

$a = 1$

$r = \frac{1}{2}$

$a = 1$

$r = \frac{1}{2}$

$= \dfrac{1}{1 - \frac{1}{2}} = 2$

1 $\frac{1}{2}$

$\frac{1}{2}$

$\frac{1}{2}$

$\frac{1}{2}$

$a + ar + ar^2 + ar^3 + ar^4 + \cdots + \infty$

If $-1 < r < 1$,

the geometric series converges.

$\text{Sum} = \dfrac{a}{1-r}$

Consider any subset of S (A)

$\text{sum}(A) \leq 2$

$A \subseteq S$

$\text{sum}(A) \leq \text{sum}(S)$

probability

0 — 1

$P(A) = \dfrac{\text{size}(A)}{\text{size}(S)} = \dfrac{\text{Sum}(A)}{\text{Sum}(S)}$

Q: Is S equilikely?

- $P(\{1\}) = \frac{1}{2}$
- $P(\{2\}) = \dfrac{\frac{1}{2}}{2} = \frac{1}{4}$

Not equilikely.

- $A = \{2, 5\}$

$\text{size}(A) = \text{Sum}\left(\frac{1}{2}, \frac{1}{16}\right)$

$= \frac{1}{2} + \frac{1}{16}$

$A \subseteq S$

sum sum

$P(A) = \dfrac{\frac{1}{2} + \frac{1}{16}}{2}$

$\underbrace{1+1+\cdots+1}_{1\text{ lac}} = 1\text{ lac}$

∞

$r = 2$

$1 + 2 + 4 + 8 + 16 + \cdots + \infty = \infty$

$r = 1$ $1 + 1 + 1 + 1 + \cdots = \frac{1}{1-1} = \frac{1}{0} = \infty$

$1 + 1 = 2$

$1 + 1 + 1 = 3$

$1 + 1 + 1 + 1 = 4$

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$

$r = \frac{1}{2}$

approaching zero

$-1 < r < 1$