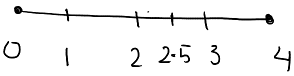
Computing prob. via counting does not work for non-simple sample spaces.

└ Infinite, or
└ Non-equilikely

① Computing P via length

$S = [0,4] \leftarrow$ all real numbers from 0 till 4 (both included)

- Uncountable
- Infinite
- Bounded ←
- Equilikely



(0, 4)
[0, 4)
(0, 4]

$A = [1, 2.5]$ is the event that selected point is in $[1, 2.5]$.

$$P(A) = \frac{\text{size}(A)}{\text{size}(S)} = \frac{\text{length}(A)}{\text{length}(S)} = \frac{2.5-1}{4-0} = \frac{1.5}{4} = 0.375$$

$0 \le P(A) \le 1$ since $\text{length}(A) \ge 0$ and $\text{length}(A) \le \text{length}(S)$ for any subset $A \subseteq S$.

$A = [3,3] = [3]$

$$P(A) = \frac{\text{length}(A)}{\text{length}(S)} = \frac{3-3}{4-0} = \frac{0}{4} = 0$$

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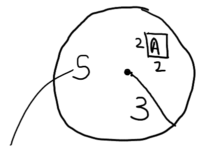
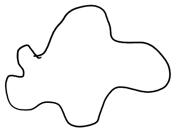
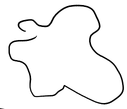
② P via area



- Bounded
- Infinite
- Equilikely

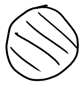
$$P(A) = \frac{\text{size}(A)}{\text{size}(S)} = \frac{\text{area}(A)}{\text{area}(S)} = \frac{4}{9\pi} = 0.141$$

$0 \le P(A) \le 1$, since $\text{area}(A) \ge 0$ and $\text{area}(A) \le \text{area}(S)$.
for any $A \subseteq S$

→ If S =  and A = , even then P(A) = area() / area()

1D, 2D, 3D, > 3D

[0, 4]



Measured size by: length, area, volume, hyper-volume

$\boxed{\text{P via measurement}}$

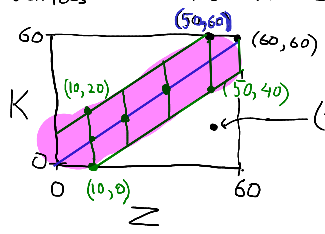
Q. Zaid and Khalid agreed to meet at a park between 3pm & 4pm. Both decided not to wait for the other person more than 10 minutes. Both arrive independently of each other at <u>random times</u> between 3 & 4 pm. What is the prob. that they will meet?

Ans.

S = set of all <u>pairs</u> of possible arrival times

Let Z be Zaid's arrival time

Let K be Khalid's arrival time



(50, 20) ⇒ Zaid arrived 50 minutes after 3pm
Khalid —— 20 ——

So, they will not meet.

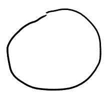
E = Purple region represents <u>all pairs</u> of arrival times for which Zaid and Khalid will meet.

S = the whole square of <u>all possible</u> pairs of arrival times.

$$P(E) = P(\text{meeting}) = \frac{\text{size}(E)}{\text{size}(S)} = \frac{\text{area}(E)}{\text{area}(S)} = \frac{3600 - \left(2\times\frac{1}{2}\times 50\times 50\right)}{3600} = \frac{3600-2500}{3600} = \frac{1100}{3600} = \frac{11}{36}$$

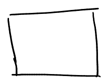
Bounded infinite

Length



Bounded infinite

Area

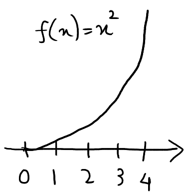


Bounded infinite

Area

S = [0,4] but chances of a number increase as we move away from 0 and towards 4.

A <u>bounded</u>, <u>infinite</u>, <u>non-equilikely</u> sample space



Unbounded infinite

$S = \{1, 2, 3, \ldots, \infty\}$

$\downarrow \quad \downarrow \quad \downarrow$

$1 \quad \frac{1}{2} \quad \frac{1}{4} \quad \ldots$

$\text{size}(S) = 1 + \frac{1}{2} + \frac{1}{4} + \cdots = 2$

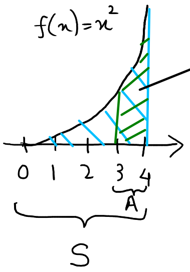
$\text{size}(A) \implies P(A)$

Artificially "assigned" numbers to elements of S

To each element x of S, we will "assign" $f(x)$.

$$\begin{array}{ccccccc} 0 & 0.1 & \ldots & 2 & \ldots & x \\ \downarrow & \downarrow & & \downarrow & & \downarrow \\ f(0) & f(0.1) & & f(2) & & f(x) \\ 0 & 0.01 & & 4 & & \end{array}$$

We will "assign" an <u>area</u> to S.



size(S) = this area

= area under the curve $f(x) = x^2$

English → from 0 till 4.

Math → $= \int_0^4 x^2\,dx$

$A = [3, 4]$

$\text{size}(A) = \int_3^4 x^2\,dx$ = area under the function x^2 from 3 till 4.

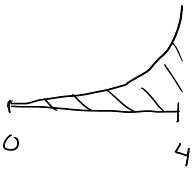
$$P(A) = \frac{\text{size}(A)}{\text{size}(S)} = \frac{\text{area}(A)}{\text{area}(S)} = \frac{\int_3^4 x^2\,dx}{\int_0^4 x^2\,dx} = \frac{\text{generalized area of } A}{\text{generalized area of } S}$$

artificially assigned

$f(x) = x^2$

$g(x) = x^4$

$\frac{37}{64} = 0.5781$



← ~~Length~~

$A_1 = [0, 1]$ $A_2 = [3, 4]$

$\text{length}(A_1) = 1 - 0 = 1$ $\text{length}(A_2) = 4 - 3 = 1$

generalized area $(A_1) =$

gen. area $(A_2) =$