Name: $\qquad$ Roll Number: $\qquad$

1. (5 points) For a symmetric, positive-definite matrix $\mathbf{A}$, show that the non-trivial maximizer of $\mathbf{x}^{T} \mathbf{A} \mathbf{x}$ is the eigenvector of $\mathbf{A}$ corresponding to the largest eigenvalue.
2. (1 point) Find the convolution mask that is convolved with the left image to give the results on the right?

| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |$* ?$|  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |$=$| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | -1 | -1 | 0 | 0 | 1 | 1 | 0 |
| 0 | -1 | -1 | 0 | 0 | 1 | 1 | 0 |
| 0 | -1 | -1 | 0 | 0 | 1 | 1 | 0 |
| 0 | -1 | -1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

3. (2 points) Write the separable versions of the following filters.
a)

$\frac{1}{16}$| 1 | 2 | 1 |
| :---: | :---: | :---: |
| 2 | 4 | 2 |
| 1 | 2 | 1 |

b)

$\frac{1}{8}$| -1 | 0 | 1 |
| :--- | :--- | :--- |
| -2 | 0 | 2 |
| -1 | 0 | 1 |

4. Let $\mathbf{I}$ be a $p \times q$ image and $\mathbf{M}$ an $m \times m$ convolution mask.
(a) (1 point) What is the cost of convolution?
(b) (1 point) What is the cost of convolution if $\mathbf{M}$ is separable?
