

# CS 565 Computer Vision – Assignment 4

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**Due Date:** Wednesday, 18th June, 2014 before class.

## Homogeneous Coordinates

Let  $\mathbf{m}_1 = (x_1, y_1)^T$  and  $\mathbf{m}_2 = (x_2, y_2)^T$  be two different points (i.e.  $\mathbf{m}_1 \neq \mathbf{m}_2$ ) in Euclidean coordinates, and let  $\ell_1 = (a_1, b_1, c_1)^T$  and  $\ell_2 = (a_2, b_2, c_2)^T$  be non-parallel lines (e.g.  $\ell_1$  is defined by  $a_1x + b_1y + c_1 = 0$ ).

1. Show that  $\mathbf{m}_1$  lies on  $\ell_1$  if and only if  $\tilde{\mathbf{m}}_1^T \ell_1 = 0$ .
2. Let  $\mathbf{m}_1$  be the intersection of  $\ell_1$  and  $\ell_2$ . Show that  $\ell_1 \times \ell_2 = \tilde{\mathbf{m}}_1$  holds.
3. Let  $\ell_1$  be the line that connects  $\mathbf{m}_1$  and  $\mathbf{m}_2$ . Show that  $\tilde{\mathbf{m}}_1^T \times \tilde{\mathbf{m}}_2 = \ell_1$ .

**Remarks:** For a point  $\mathbf{x} = (x_1, x_2)^T$  in Euclidean coordinates, the point  $\tilde{\mathbf{x}} = (x_1, x_2, 1)^T$  is its counterpart in homogeneous coordinates. The operator  $\times$  denotes the cross product

$$\mathbf{x} \times \mathbf{y} := \begin{bmatrix} x_2y_3 - x_3y_2 \\ x_3y_1 - y_1x_3 \\ x_1y_2 - x_2y_1 \end{bmatrix} \quad (\mathbf{x}, \mathbf{y} \in \mathbb{R}^3)$$

## 3-D Rotation and Euler Angles

A sequence of three rotations can be used to give a 3-D-object a specific orientation. To this end we first rotate  $\Psi$  degrees around the  $z$ -axis. This is followed by a rotation around the **new  $x$ -axis** by an angle of  $\Theta$ . Finally we rotate  $\Phi$  degrees around the  **$z$ -axis that was created by the previous rotations**. State the final rotation matrix that depends only on  $\Psi, \Theta$ , and  $\Phi$ .

## Transformation Matrices

Compute a transformation matrix in homogeneous coordinates which describes a rotation around the  $y$ -axis through an angle of 45 degrees followed by a translation with a vector  $(-1, 2, -3)^T$  and a rotation around the  $x$ -axis through an angle of  $-60$  degrees.

## Fundamental Matrix for the Orthoparallel Camera Setup

For the orthoparallel case, the fundamental matrix  $\mathbf{F}$  is uniquely defined up to a scaling factor. Determine the fundamental matrix for the orthoparallel case.

**Hint:** Exploit the formula  $\ell_2 = \mathbf{F}\tilde{\mathbf{m}}_1$ , using the 3 homogeneous points  $(0, 0, 1)^T$ ,  $(1, 0, 1)^T$  and  $(0, 1, 1)^T$  and their corresponding lines. Recall that the vector  $\ell_2 = (a, b, c)^T$  describes the epipolar line  $ax + by + c = 0$  in the second frame which corresponds to a given point  $\mathbf{m}_1$  in the first frame.

## Stereo Reconstruction

Let us assume that we have two cameras in  $\mathbf{C}_1$  and  $\mathbf{C}_2$ . The extrinsic and intrinsic parameters of both cameras are given in the form of the matrices

$$\mathbf{A}_1^{\text{int}} = \mathbf{A}_2^{\text{int}} = \begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$\mathbf{A}_1^{\text{ext}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{A}_2^{\text{ext}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & -5 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Moreover, the focal length of both cameras is given by  $f_1 = f_2 = 1$ . Let us assume, that we have found the following correspondence:

$$\mathbf{x}_1 = \left( \frac{21}{2}, \frac{1}{2} \right)^T, \quad \mathbf{x}_2 = \left( \frac{19}{2}, \frac{\sqrt{2}}{2} \right)^T.$$

Here,  $\mathbf{x}_1$  and  $\mathbf{x}_2$  denote a point in pixel locations in the first and in the second frame, respectively. In order to restore the depth of the original scene point, perform the following steps:

1. Compute the corresponding optical rays in the 3-D coordinate system of the cameras.
2. Compute the corresponding optical rays in the 3-D Euclidean world coordinate system.
3. Intersect these rays to recover the 3-D world coordinates of the 3-D point that was projected on both image planes.

## Submission

Submit your hand-written assignment **before class starts** to your TA Naila Hamid (naila.hamid@pucit.edu.pk).

**Note:** To submit your results in a beautiful looking .pdf file, the LaTeX source for this document is also provided in the Assignment4.tex file. You can use the command `\answer{}` to fill in your answers below each question. Please consult your TA for more help. **Remember: Word is ugly and LaTeX is beautiful!**