MA-250 Probability and Statistics

Nazar Khan

PUCIT

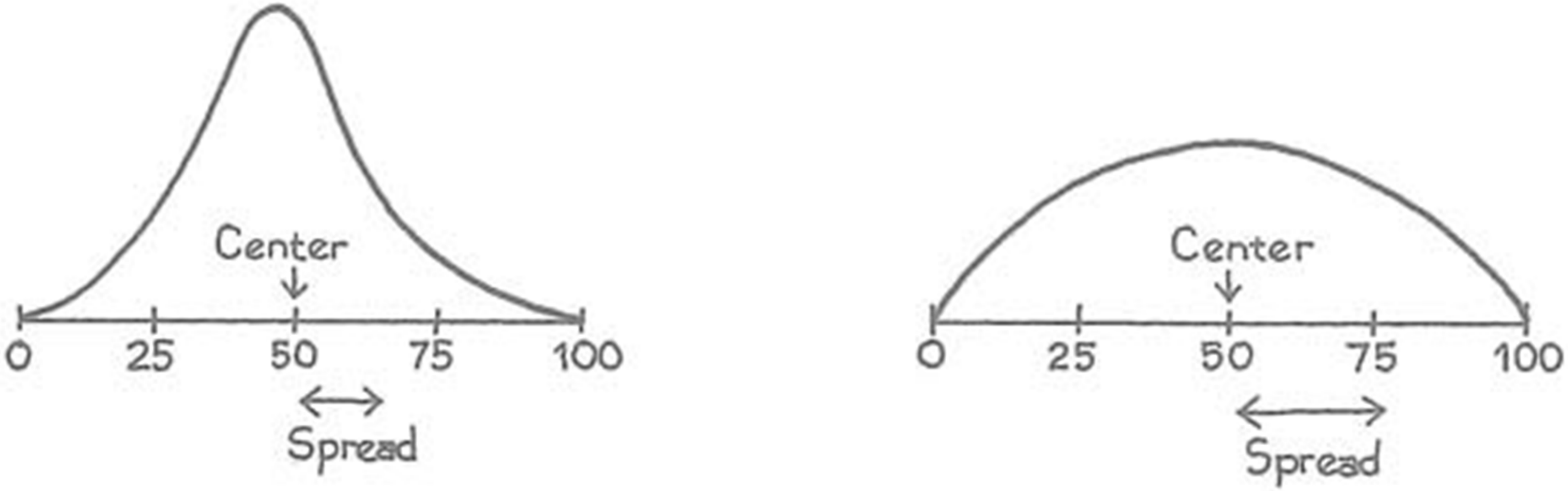
Lecture 3

Average and Standard Deviation

- A histogram tries to summarize large amounts of data.
- An even more drastic summary can be given by the histogram's
  - Center
  - Spread

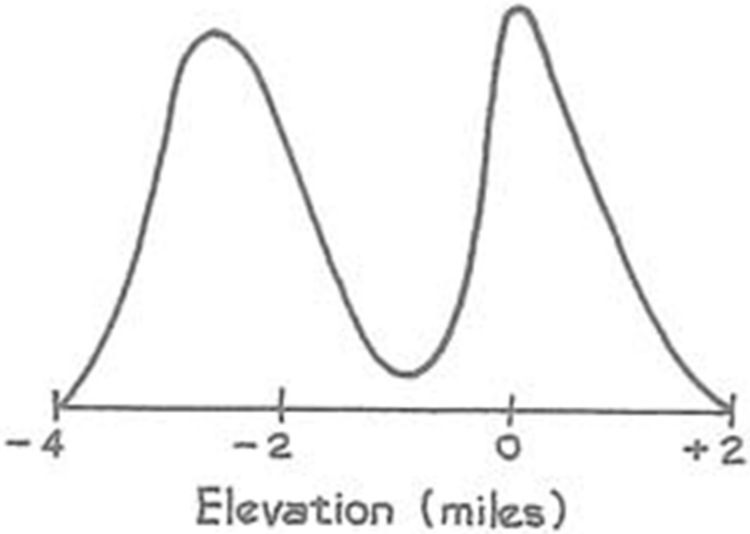
Average and Spread

Figure 1. Center and spread. The centers of the two histograms are the same, but the second histogram is more spread out.

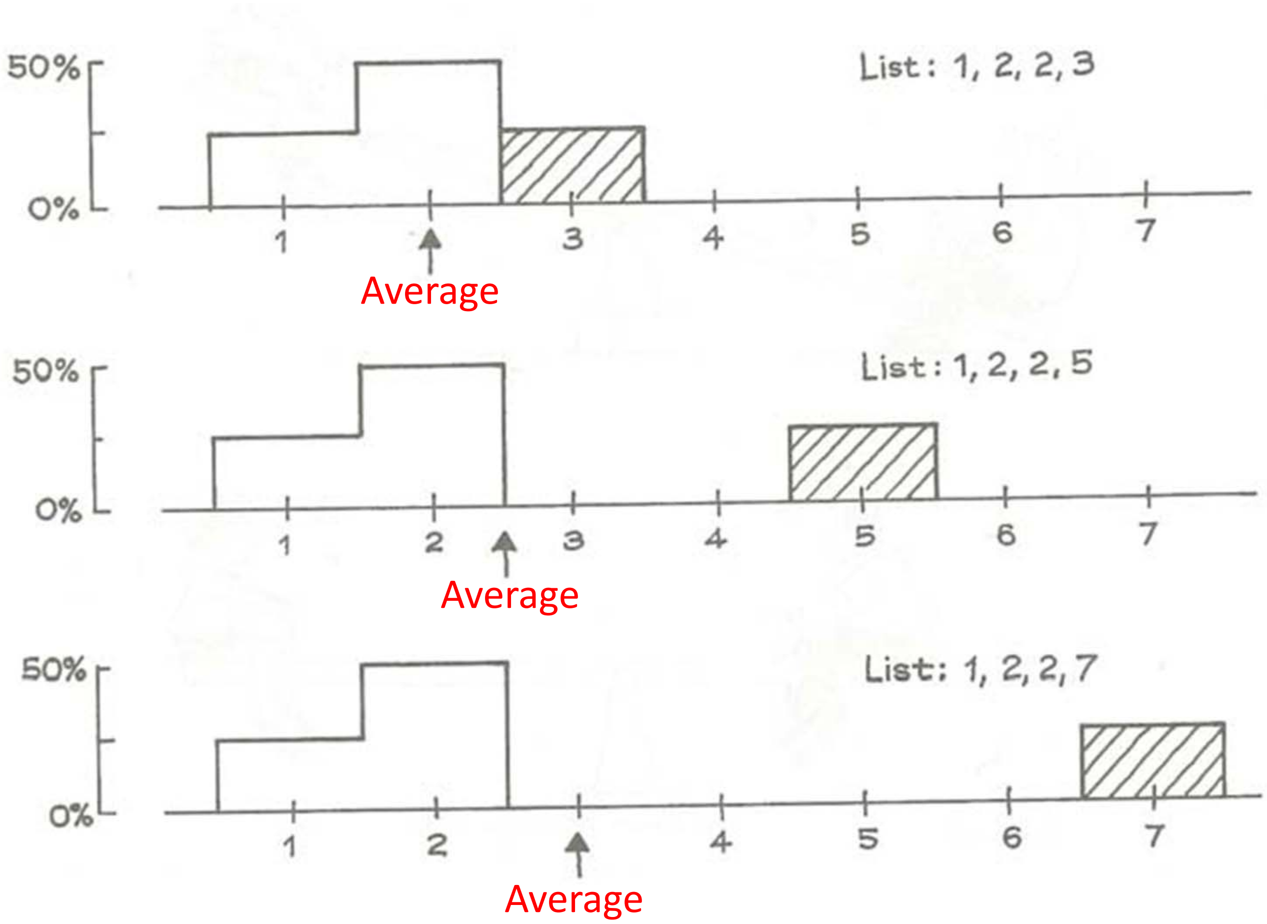


But not always…

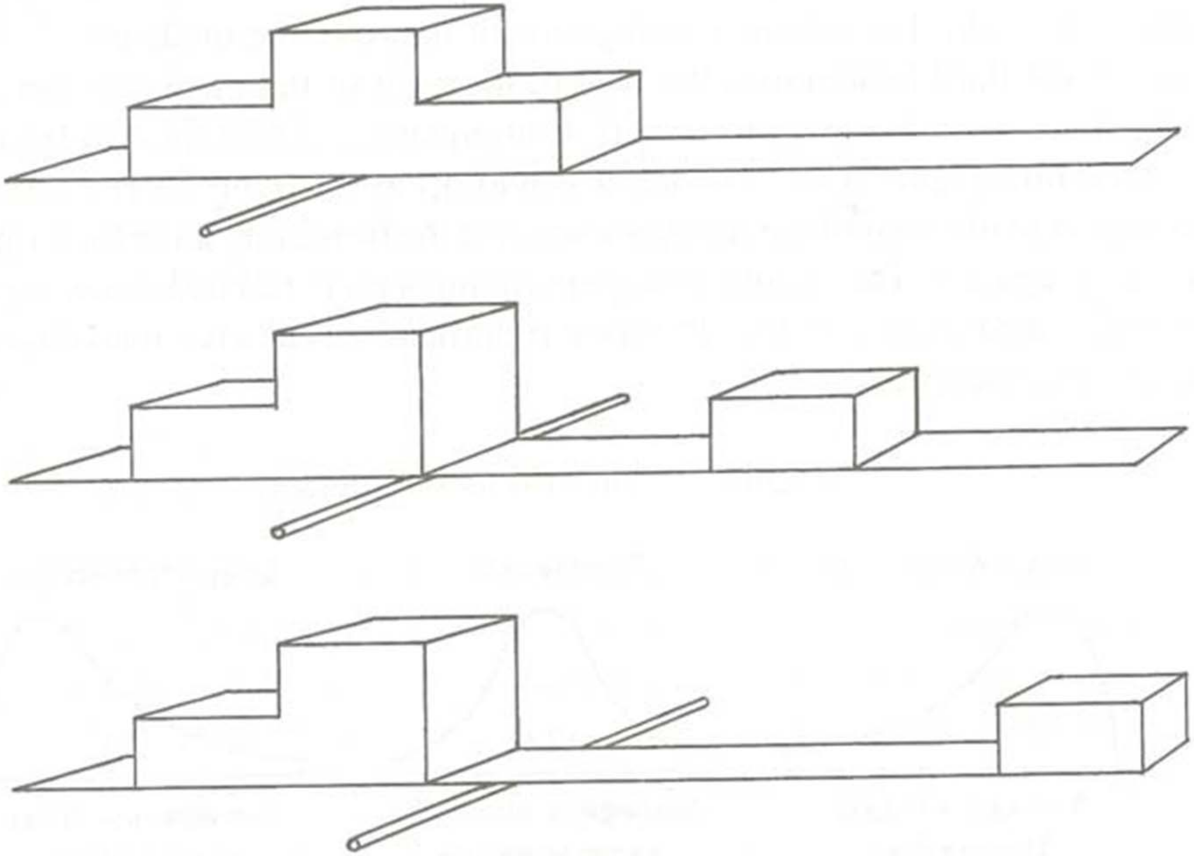
Figure 2. Distribution of the surface area of the earth by elevation above (+) or below (–) sea level.



Average balances the histogram

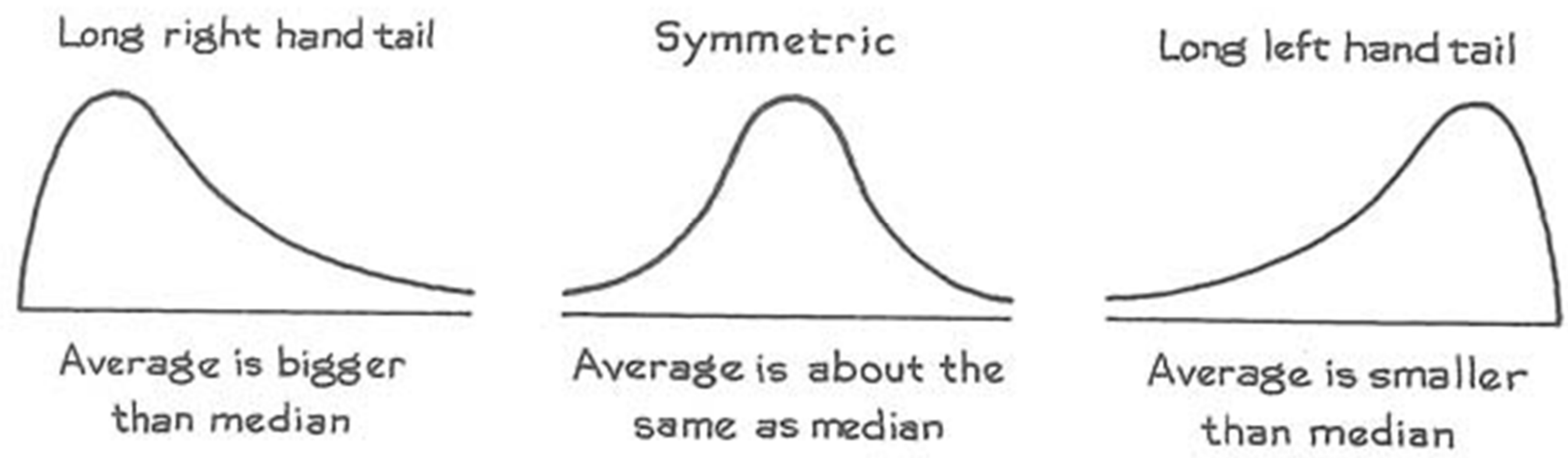


Average balances the histogram



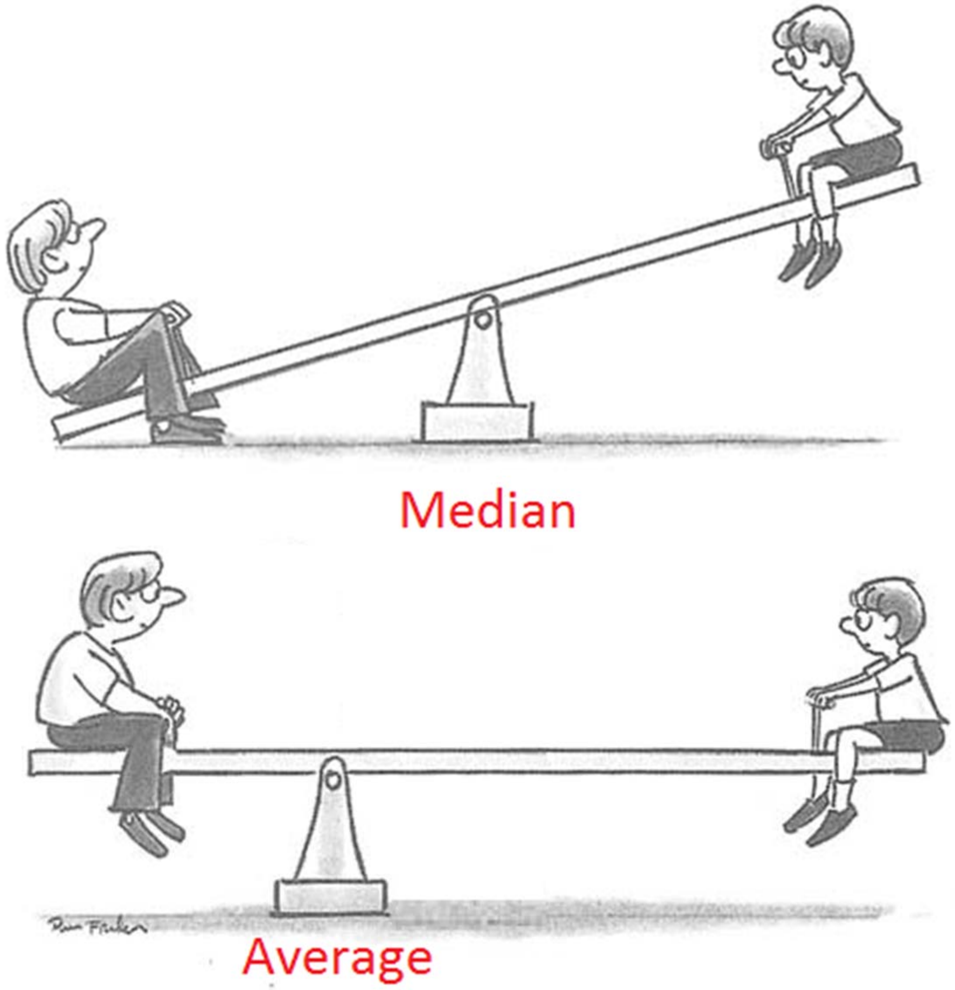
Median

- Median of a **histogram** is the value with half the area to the left and half to the right.


Long right hand tail
Average is bigger than median
Symmetric
Average is about the same as median
Long left hand tail
Average is smaller than median

Median

Median of a **list** is the value from which half or more values are larger and half or more are smaller.



Lies in the middle

Balances both sides

Median

- Compute median of
  - 2,6,8
  - 4,8,9,13
  - 1,2,2,7,8
  - 8,-3,5,0,1,4,-1
  - 800,-3,5,0,1,4,-1

Average vs. Median

- Which estimate is better when data contains outliers?
  - Median since it is not affected by outliers.

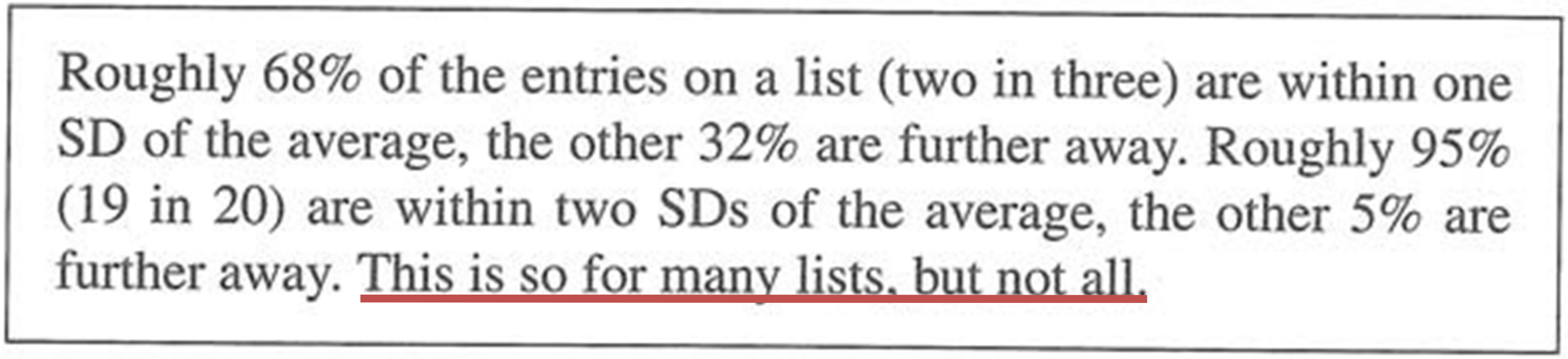
	List 1	**List 2**
	1	1
	2	2
	3	3
	4	4
	5	5
	6	6
	7	7
	8	8
	9	9
	10	100
Average	5.5	14.5
Median	5.5	5.5

Outlier

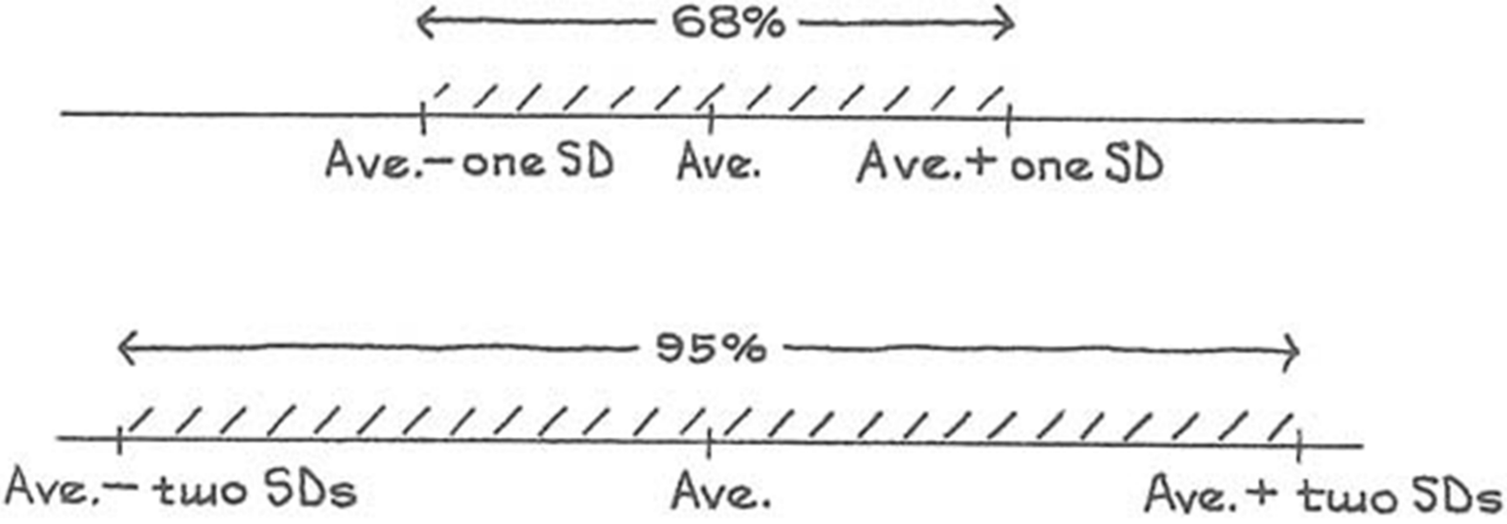
Measuring Spread – Standard Deviation

- It is usually quite helpful to see how a list of numbers spreads around the average value.
- This is measured by the **standard deviation (SD)**.
- SD = root-mean-square (r.m.s) deviation from average
- Compute SD of 20,10,10,15
  1. Compute average
  2. Compute deviations from average
  3. Compute r.m.s of deviations

Magic of Standard Deviation
The 68-95-99 Rule

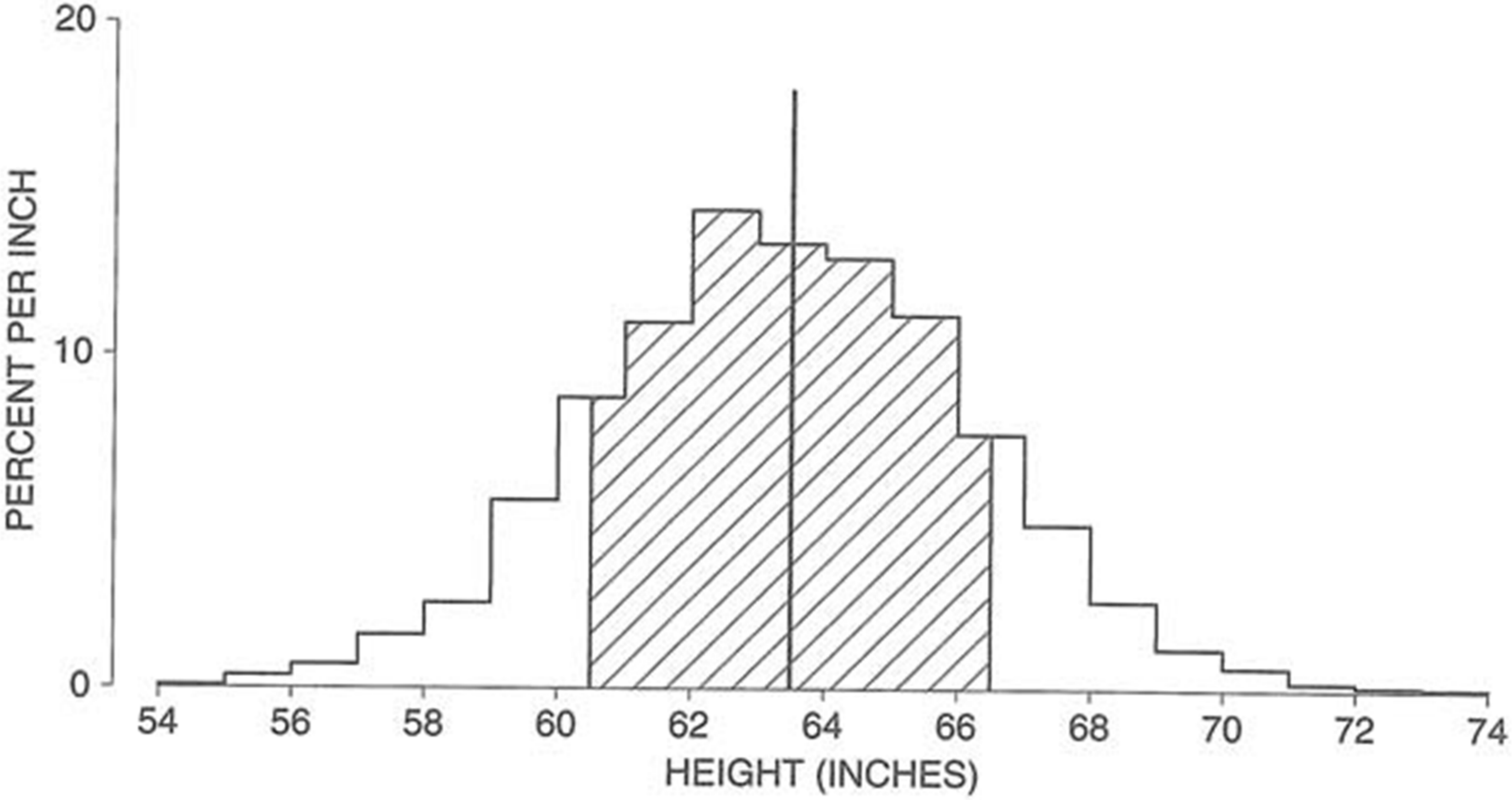


Roughly 68% of the entries on a list (two in three) are within one SD of the average, the other 32% are further away. Roughly 95% (19 in 20) are within two SDs of the average, the other 5% are further away. This is so for many lists, but not all.



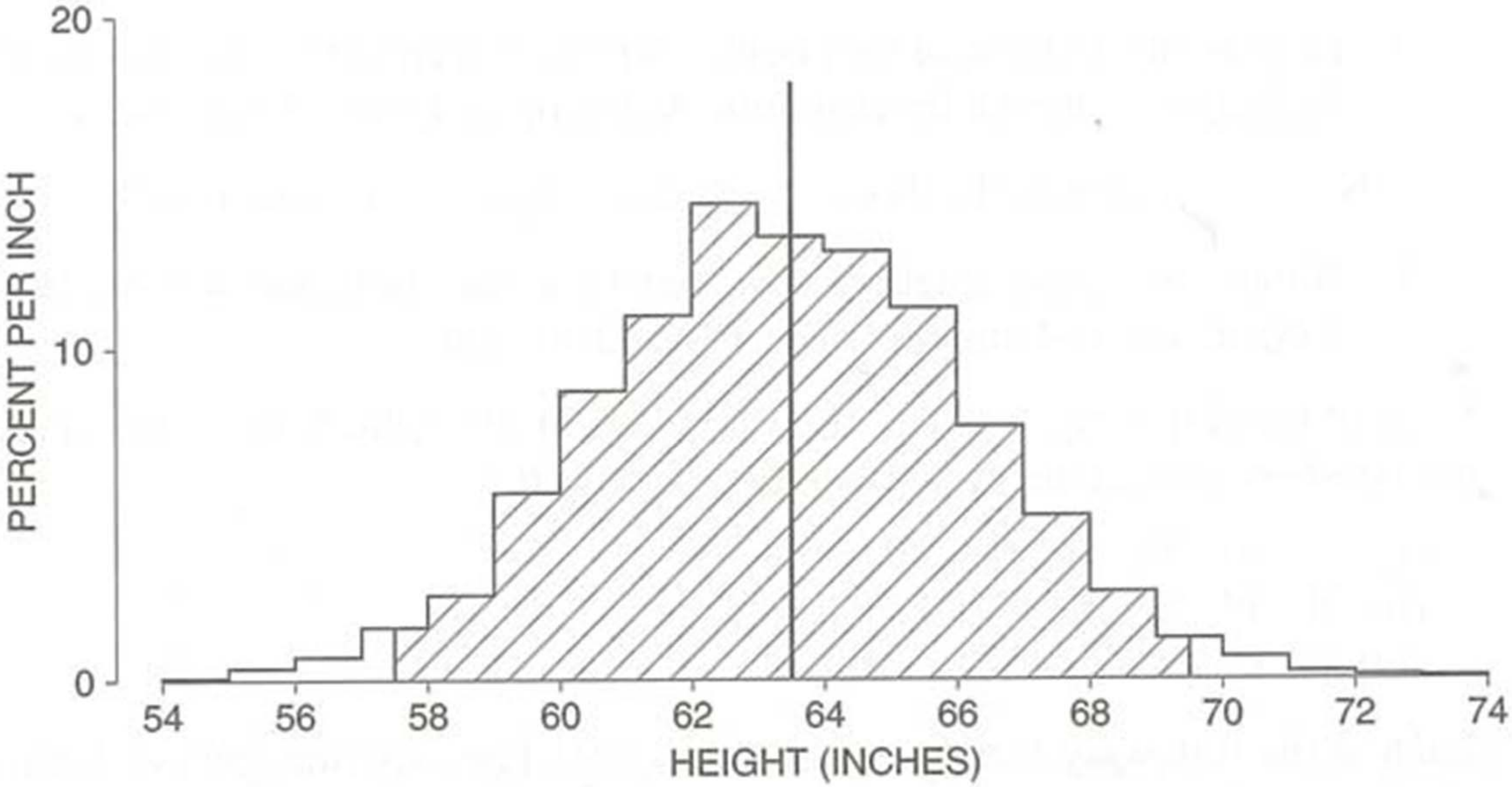
The 68-95-99 Rule

Figure 8. The SD and the histogram. Heights of 2,696 women age 18 and over in HANES5. The average of 63.5 inches is marked by a vertical line. The region within one SD of the average is shaded: 72% of the women differed from average by one SD (3 inches) or less.

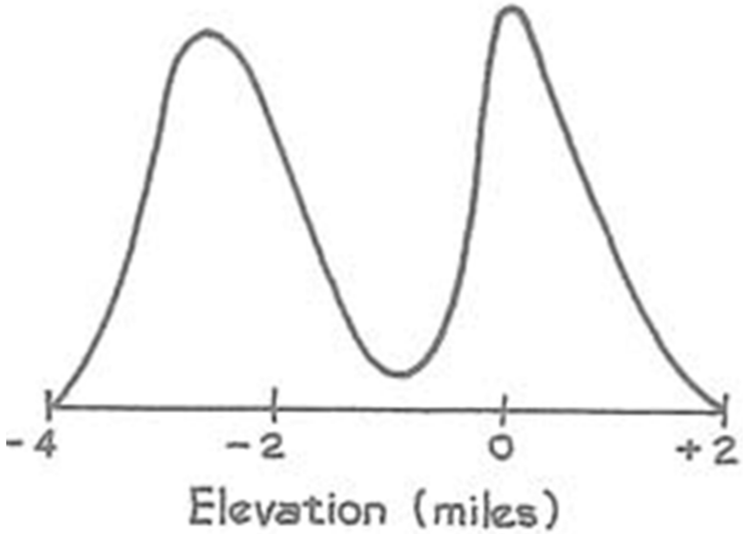


The 68-95-99 Rule

Figure 9. The SD and the histogram. Heights of 2,696 women age 18 and over in HANES5. The average of 63.5 inches is marked by a vertical line. The region within two SDs of the average is shaded: 97% of the women differed from average by two SDs (6 inches) or less.



Not Always …



Summary

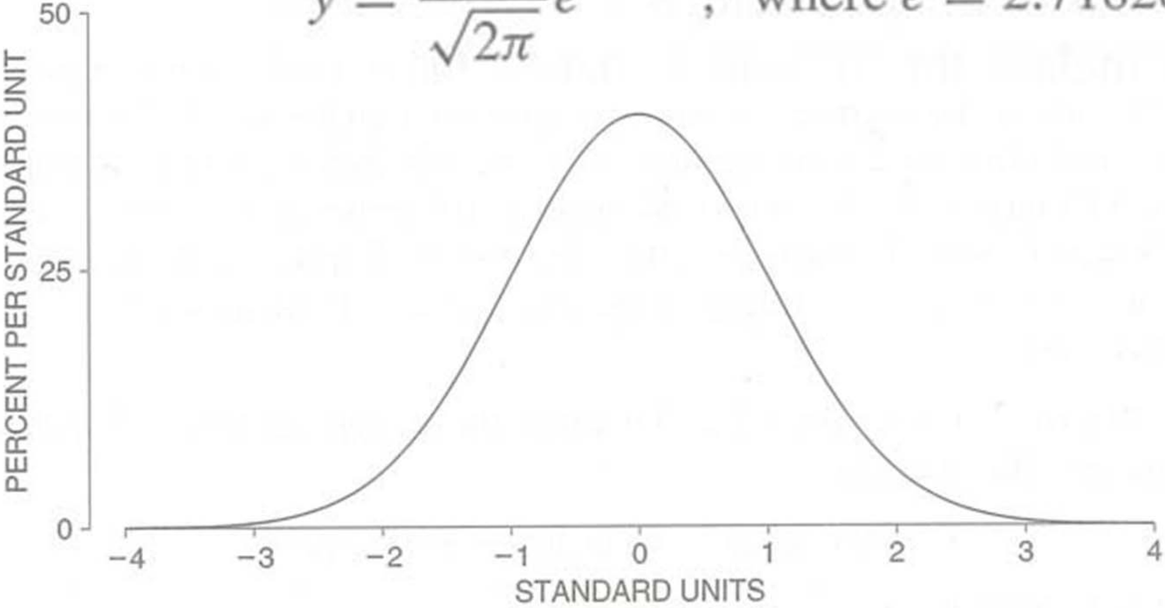
- Usually a list of numbers can be well-summarized by its average and standard deviation
- Center of histogram
  - Average – balances the histogram
  - Median – divides histogram areas into half
- Standard deviation measures spread around the average
- Usually
  - Approximately 68% data lies within 1 SD of the average
  - Approximately 95% data lies within 2 SD of the average
  - Approximately 99% data lies within 3 SD of the average

The Normal Curve

- An approximation to data distribution that is **normally** quite accurate
  - Normally data follows such a distribution

The Normal Curve

$$y = \frac{100\%}{\sqrt{2\pi}} e^{-x^2/2}, \quad \text{where } e = 2.71828$$



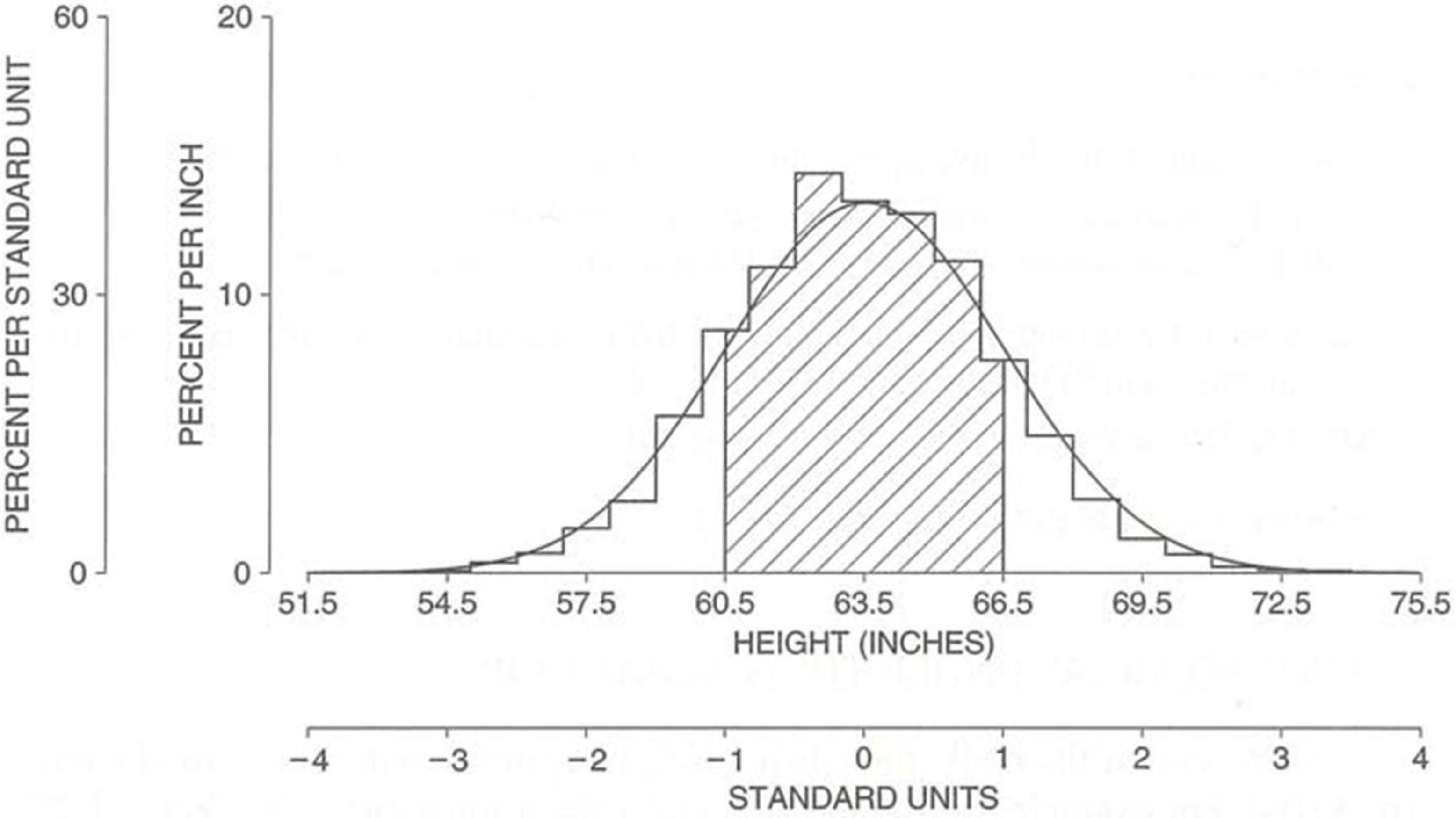
- the area under the normal curve between −1 and +1 is about 68%;
- the area under the normal curve between −2 and +2 is about 95%;
- the area under the normal curve between −3 and +3 is about 99.7%.

Standard Units

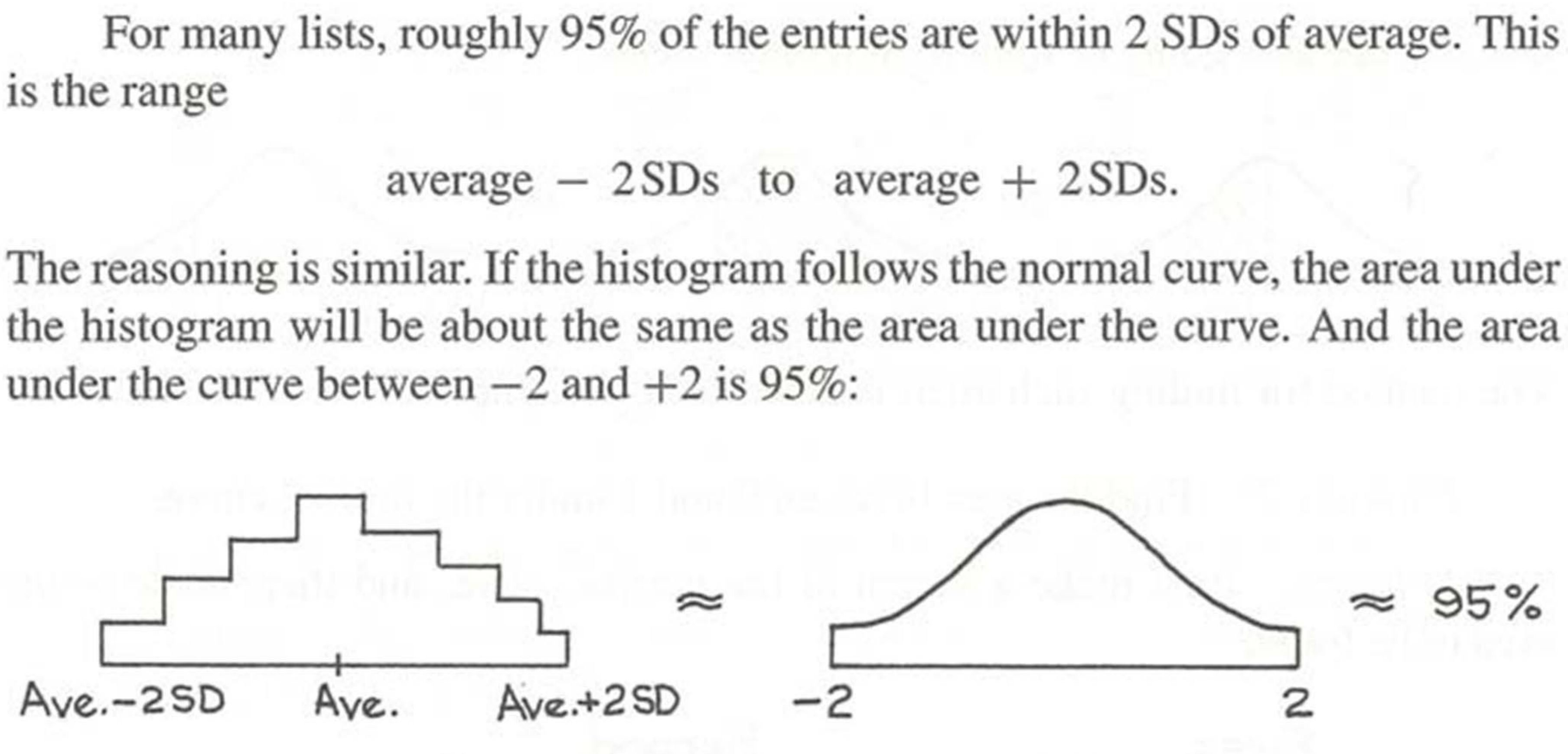
- Express the data in terms of standard deviation
- Converting a value X to standard units
  - (X-average)/SD

The Normal Approximation to Data

Figure 2. A histogram for heights of women compared to the normal curve. The area under the histogram between 60.5 inches and 66.5 inches (the percentage of women within one SD of average with respect to height) is about equal to the area between −1 and +1 under the curve—68%.



The Normal Approximation to Data



For many lists, roughly 95% of the entries are within 2 SDs of average. This is the range

$$\text{average} - 2\text{SDs} \quad \text{to} \quad \text{average} + 2\text{SDs}.$$

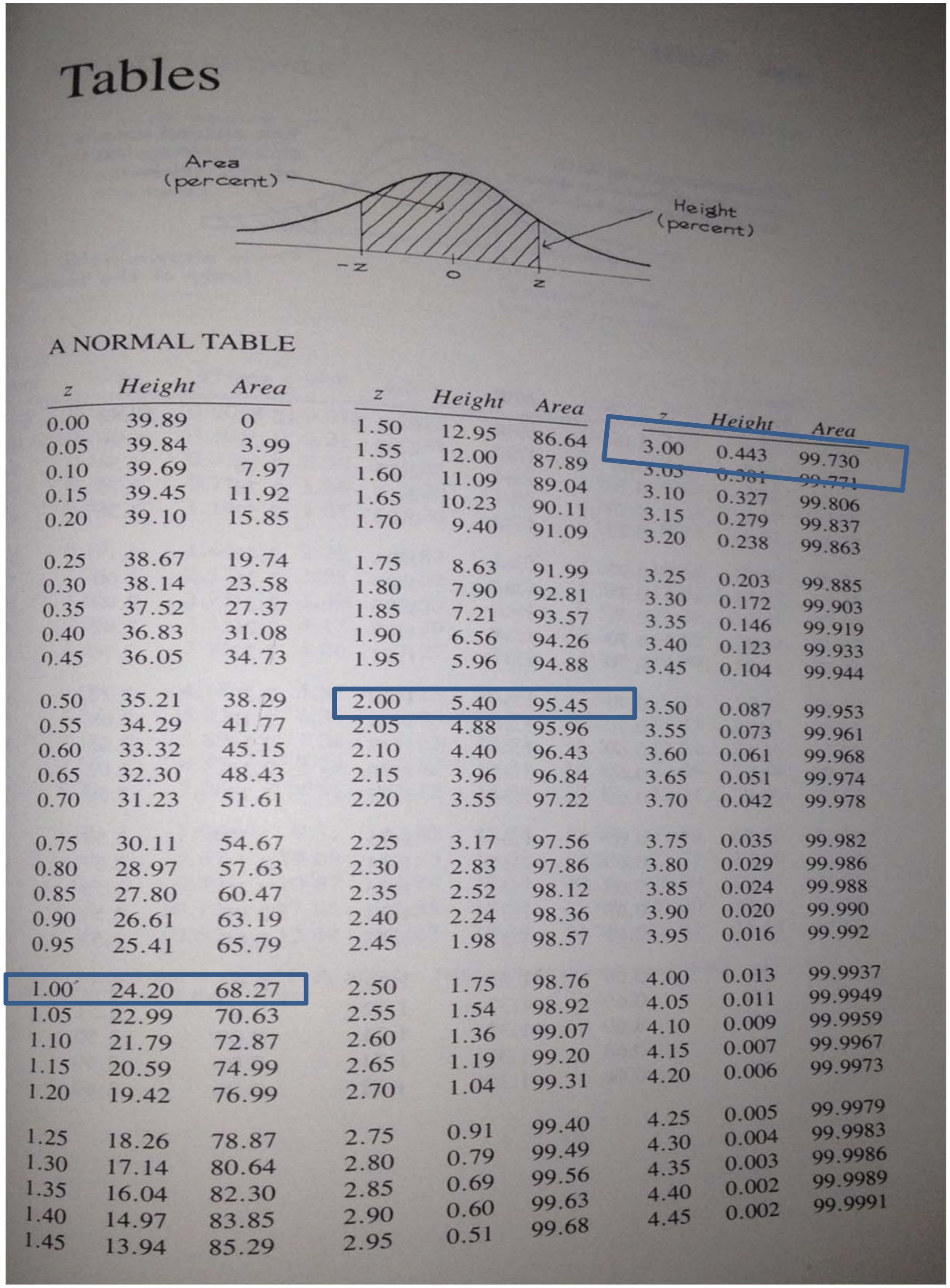
The reasoning is similar. If the histogram follows the normal curve, the area under the histogram will be about the same as the area under the curve. And the area under the curve between −2 and +2 is 95%:

Ave.−2SD
Ave.
Ave.+2SD
≈
−2
2
≈ 95%

On a certain exam, the average of the scores was 50 and the SD was 10.

(a) Convert each of the following scores to standard units: 60, 45, 75.

(b) Find the scores which in standard units are: 0, $+1.5$, -2.8.



Tables

Area
(percent)
Height
(percent)
−z
0
z

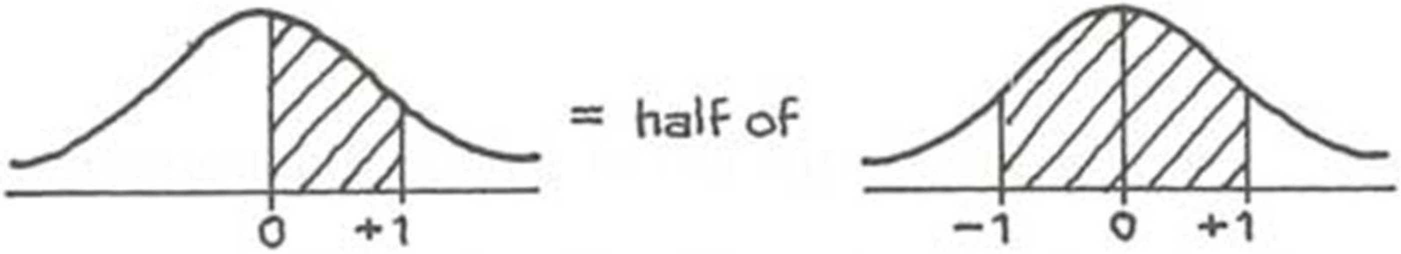
A NORMAL TABLE

z	*Height*	*Area*	*z*	*Height*	*Area*	*z*	*Height*	*Area*
0.00	39.89	0	1.50	12.95	86.64	3.00	0.443	99.730
0.05	39.84	3.99	1.55	12.00	87.89	3.05	0.381	99.771
0.10	39.69	7.97	1.60	11.09	89.04	3.10	0.327	99.806
0.15	39.45	11.92	1.65	10.23	90.11	3.15	0.279	99.837
0.20	39.10	15.85	1.70	9.40	91.09	3.20	0.238	99.863
0.25	38.67	19.74	1.75	8.63	91.99	3.25	0.203	99.885
0.30	38.14	23.58	1.80	7.90	92.81	3.30	0.172	99.903
0.35	37.52	27.37	1.85	7.21	93.57	3.35	0.146	99.919
0.40	36.83	31.08	1.90	6.56	94.26	3.40	0.123	99.933
0.45	36.05	34.73	1.95	5.96	94.88	3.45	0.104	99.944
0.50	35.21	38.29	2.00	5.40	95.45	3.50	0.087	99.953
0.55	34.29	41.77	2.05	4.88	95.96	3.55	0.073	99.961
0.60	33.32	45.15	2.10	4.40	96.43	3.60	0.061	99.968
0.65	32.30	48.43	2.15	3.96	96.84	3.65	0.051	99.974
0.70	31.23	51.61	2.20	3.55	97.22	3.70	0.042	99.978
0.75	30.11	54.67	2.25	3.17	97.56	3.75	0.035	99.982
0.80	28.97	57.63	2.30	2.83	97.86	3.80	0.029	99.986
0.85	27.80	60.47	2.35	2.52	98.12	3.85	0.024	99.988
0.90	26.61	63.19	2.40	2.24	98.36	3.90	0.020	99.990
0.95	25.41	65.79	2.45	1.98	98.57	3.95	0.016	99.992
1.00	24.20	68.27	2.50	1.75	98.76	4.00	0.013	99.9937
1.05	22.99	70.63	2.55	1.54	98.92	4.05	0.011	99.9949
1.10	21.79	72.87	2.60	1.36	99.07	4.10	0.009	99.9959
1.15	20.59	74.99	2.65	1.19	99.20	4.15	0.007	99.9967
1.20	19.42	76.99	2.70	1.04	99.31	4.20	0.006	99.9973
1.25	18.26	78.87	2.75	0.91	99.40	4.25	0.005	99.9979
1.30	17.14	80.64	2.80	0.79	99.49	4.30	0.004	99.9983
1.35	16.04	82.30	2.85	0.69	99.56	4.35	0.003	99.9986
1.40	14.97	83.85	2.90	0.60	99.63	4.40	0.002	99.9989
1.45	13.94	85.29	2.95	0.51	99.68	4.45	0.002	99.9991

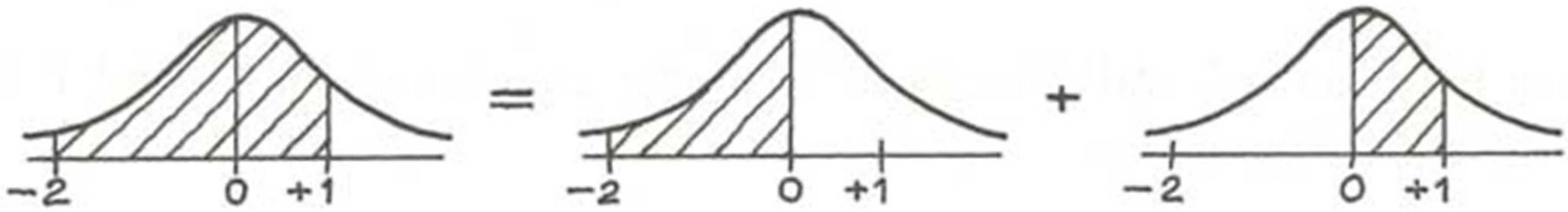
Finding Areas under Normal Curve

Find the area between 0 and 1 under the normal curve.

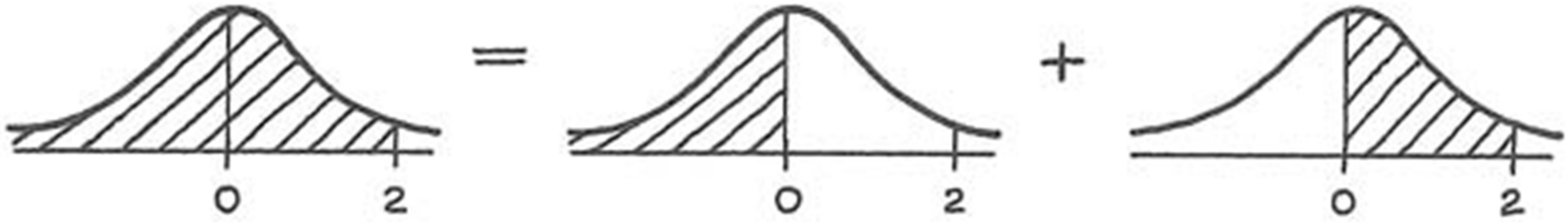
$$\frac{1}{2} \times 68\% = 34\%$$



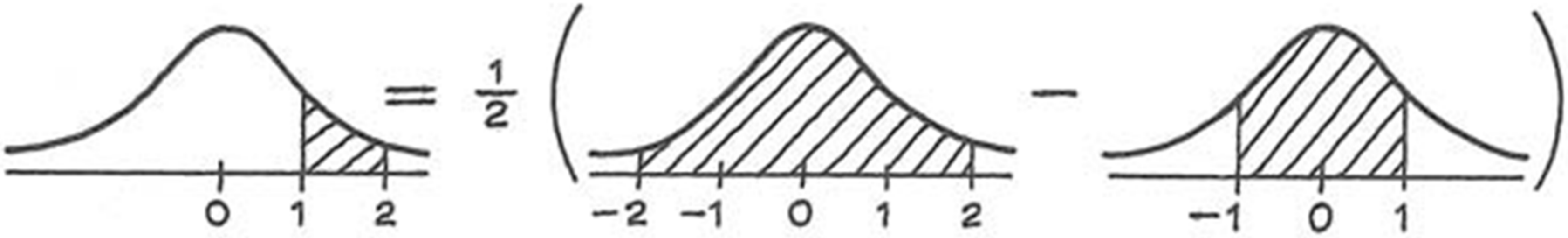
- Find the area between 0 and 2 under the normal curve.
- Find the area between -2 and 1 under the normal curve.



- Find the area to the right of 1 under the normal curve.
- Find the area to the left of 2 under the normal curve.



- Find the area between 1 and 2 under the normal curve



Example 8. The heights of the men age 18 and over in HANES5 averaged 69 inches; the SD was 3 inches. Use the normal curve to estimate the percentage of these men with heights between 63 inches and 72 inches.