Modelling Trust for Communicating Agents: Agent-Based and Population-Based Perspectives

S. Waqar Jaffry, Jan Treur

VU University Amsterdam, Department of Artificial Intelligence
De Boelelaan 1081, 1081 HV Amsterdam, The Netherlands
Email: { swjaffry, treur}@few.vu.nl
URL: http://www.few.vu.nl/~{ swjaffry, treur}

Abstract.

This paper presents an exploration of the differences between agent-based and population-based models for trust dynamics. This exploration is based on both a large variety of simulation experiments and a mathematical analysis of the equilibria of the two types of models. The outcomes show that the differences between the models are not very substantial, and become less for larger numbers of agents.

Keywords: agent-based, population-based, trust

1 Introduction

When a population of agents is considered, the dynamics of trust in a certain trustee can be modelled from two perspectives: from the agent-based perspective and from the population-based perspective. From the agent-based perspective each agent has its own characteristics and maintains its own trust level over time. From the population-based perspective one trust level for the whole is maintained over time, depending on characteristics of the population. For both cases dynamical models can be used to determine the trust levels over time. For the agent-based perspective, each agent has its own dynamical model (for example, expressed as a system of *N* differential equations, with *N* the number of agents), whereas for the population-level one model (for example, expressed as one differential equation) can be used. From the agent-based model, by aggregation a collective trust level for the population as a whole can be determined, for example, by taking the average over all agents.

Usually agent-based simulation is computationally much more expensive than population-based simulation. However, this still may be worth the effort when it is assumed that the outcomes substantially differ from the outcomes of a population-based simulation. In this paper this assumption is explored in a detailed manner for a population of agents that not only receive direct experiences for a trustee but also get communicated information from other agents about their trust in this trustee. On the one hand the analysis makes use of a variety of simulation experiments for different population sizes and different distributions of characteristics. On the other hand a mathematical analysis of equilibria of both types of models is used to find out differences between the two types of models. Roughly spoken, the outcome of both types of investigations are that in general the differences are not substantial, and that they are smaller the larger the number of agents is.

In Section 2 the two types of models used are introduced. In Section 3 the simulation experiments are described. Section 4 presents the mathematical analysis of equilibria. Section 5 concludes the paper.

2 Modelling Trust Dynamics from Two Perspectives

In this section trust models for both perspectives are introduced. The basic underlying trust dynamics model adopted in this paper depends on receiving experiences $E(t)$ over time as follows:

$$T(t + \Delta t) = T(t) + \gamma * \big(E(t) - T(t)\big) * \Delta t$$

Here $T(t)$ and $E(t)$ are the trust level for a trustee and the experience level given by the trustee at time point t. Furthermore, γ is a personal characteristic for flexibility: the rate of change of trust upon receiving an experience $E(t)$. The values of $T(t)$, $E(t)$ and γ are in the interval $[0, 1]$. In differential form change of trust over time can be expressed by

$$\frac{dT}{dt} = \gamma * (E - T)$$

This basic model is based on the experienced-based trust model described in [1], and applied in [3, 4, 5]. In the case of communicating agents, experiences are taken to be of two forms: direct experiences acquired, for example, by observation, and indirect experiences, obtained from communication. Incorporating this, the basic model can be applied to each single agent within the population (agent-based perspective), or to the population as a whole (population-based perspective), as discussed below.

2.1. An Agent-Based Trust Model Incorporating Communication

In the agent-based trust model for a trustee described here, each of the agents updates its trust on a given trustee based on receiving an experience for this trustee which combines a direct experience and an opinion received by the peers about the trustee (indirect experience). Direct and indirect experiences at each time point are aggregated using agents' personality characteristic called social influence denoted by α_A as follows:

$$E_A(t) = \alpha_A * E_A^i(t) + (1 - \alpha_A) * E_A^d(t)$$

Here $E_A(t)$, $E_A{}^d(t)$ and $E_A{}^i(t)$ are the aggregated experience, the direct experience received from the trustee and the indirect experience received by the agent A as the opinions of its peers at about trustee at time t respectively.

The indirect experience $E_A^i(t)$ received by the agent A as the opinions of its peers about trustee at time point t is taken the average of the opinions given by all the peers at time point t :

$$E_A^i(t) = \sum_{B \neq A} O_B(t)/(N-1)$$

Here $O_B(t)$ is the opinion received by the agent A from an agent B about the trustee at time point t and N is the total number of agents in the population. The opinion given by the agent B to the agent A at time t is taken as the value of the trust of j on trustee at time t so,

$$O_B(t) = T_B\,(t)$$

The aggregated experience received by agent A at time point t is used to update current trust level of the agent A at trustee using trust model presented in the previous section as follows

$$T_A(t + \Delta t) = T_A(t) + \gamma_A * \big(E_A(t) - T_A(t)\big) * \Delta t$$

Here the basic trust model is indexed for each agent A in the group. Note that each agent can have its personal flexibility characteristic γ_A. It is assumed that these values have some distribution over the population.

Based on this agent-based model a collective trust value $T_C(t)$ for the population as a whole can be obtained by aggregation of the trust values over all agents (taking the average):

$$T_C(t) = \frac{1}{N} \Sigma_A T_A(t)$$

2.2. A Population-Based Trust Model Incorporating Communication

To apply the basic trust model to obtain a population-based model of trust, its ingredients have to be considered for the population P as a whole, for example, the (direct and indirect) experience given by the trustee to a population P, and the characteristics γ_P of the population [8, 10]; this is done as follows

$$T_P(t + \Delta t) = T_P(t) + \gamma_P * \big(E_P(t) - T_P(t)\big) * \Delta t$$

Here $T_P(t)$ is the trust of population P on a given trustee at time point t, and the population-level flexibility characteristic γ_P is taken as an aggregate value for the individual flexibility characteristics γ_A for all agents A in P (e.g., the average of the γ_A for $A \in P$). This can be interpreted as if the population as a whole is represented as one agent who receives experiences from the trustee and updates its trust on the trustee using the basic model. The experience at population level $E_P(t)$ at time point t for the population P is defined as ta combination of the direct and the indirect experience at population level as follows,

$$E_P(t) = \alpha_P * E_P^i(t) + (1 - \alpha_P) * E_P^d(t)$$

In the above equation $E_P^i(t)$ and $E_P^d(t)$ are the indirect and direct experience at the population level. Moreover, α_P is the population-level social influence characteristic. Here also α_P is taken as an aggregate value for the individual social influence characteristics α_A for all agents present in P (e.g., the average of the α_A for $A \in P$). At the population level the indirect experience $E_P^i(t)$ obtained from communication by the other agents of their trust is taken as the population level trust value at time point t as follows:

$$E_P^i(t) = T_P(t)$$

2.3. Complexity Estimation

The complexity of the agent-based trust model differs from that of the population-based models in the sense that for the agent-based trust model the complexity depends on the number of agents while this is not the case for the population-based model. This can be vestimated as follows. For τ the total number of time steps, and N the number of agents in the population, the time complexities of the agent-based and population-based models are $O(N^2\tau)$ and $O(\tau)$ respectively. This indicates that for higher numbers of agents in a population the agent-based model is computationally much more expensive.

3 Simulating and Comparing the Two Trust Models

A number of simulation experiments have been conducted to compare the agent-based and population-based trust models as described in the previous sections. This section presents the experimental setup and results from these experiments.

3.1. The Experimental Setup

For the simulation experiments a setup was used as shown in Fig. 1. Here a trustee S is assumed to give similar direct experiences $E^d(t)$ to both models at each time point t. In the population-based trust model this direct experience $E^d(t)$ is used together with the indirect experience $E_P^i(t)$ to update the population-level trust of S according to the equations presented in Section 2.2. In the agent-based trust model this experience is received by every agent in the system and each agent updates its trust on the trustee using direct experience $E^d(t)$ and indirect experience $E_A^i(t)$ received as opinion of the other agents, as shown in Section 2.1. By aggregation the individual trust levels can be used to obtain a collective trust of the trustee.

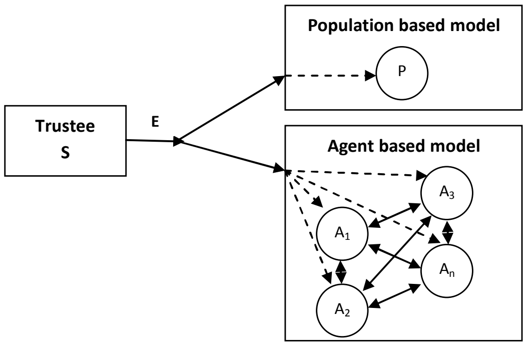


Figure 1. Agent and population based trust model

In Fig. 1. P carries the population-based trust model while the agents A_1, A_2, A_3 … A_n carry the agent-based trust model as described in the previous sections. Every agent in the system is assigned an initial trust value $T_A(0)$, a value for the agent's flexibility γ_A, and for the social influence parameter α_A at the start of the simulation experiment. The value $T_P(0)$ for the initial population-level trust, the population-level flexibility parameter γ_P and the social influence α_A parameter for the population-based trust model are taken as the average of the corresponding attributes of all the agents in the community:

$$T_P(0) = \sum_A T_A(0)/N \quad , \quad \gamma_P = \sum_A \gamma_A/N \quad , \quad \alpha_P = \sum_A \alpha_A/N$$

Here N is the total number of agents in the community. The collective trust of the agent-based trust model at any time point t is represented as the average of the trust values of all the agents in the community:

$$T_C(t) = \sum_A T_A(t)/N$$

As a measure of dissimilarity for the comparison of the models their root mean square error is measured between the collective agent-level trust and population-level trust at each time point t as follows

$$\varepsilon = \sqrt{\sum_{t=1}^{t=M} (T_P(t) - T_C(t))^2/M}$$

In the above equation $T_P(t)$ and $T_C(t)$ are the population-level trust and the (aggregated) collective agent-level trust of the trustee calculated by the population-based and agent-based model at time point t respectively and M is the total time steps in the simulation.

To produce realistic simulations, the values for $T_A(0)$, γ_A and α_A of all the agents in the agent-based trust model were taken from a uniform normal distribution with mean value *0.50* and standard deviation varying from *0.00* to *0.24*. In these experiments all agent-based models were simulated exhaustively to see their average behavior against the population-based model. Here exhaustive simulation means that all possible combinations of standard deviations for $T_A(0)$, γ_A and α_A from the interval *0.00-0.24* were used in the simulations and their respective errors were measured against respective population level model. An average error ε_{avg} of the models was calculated, which is the average of all root mean squared errors calculated with all combinations of $T_A(0)$, γ_A and α_A as follows.

$$\varepsilon_{avg} = \frac{\sum_{stDev_{TA(0)}=0.00}^{stDev_{TA(0)}=0.24}\left(\sum_{stDev_{\gamma A}=0.00}^{stDev_{\gamma A}=0.24}\left(\sum_{stDev_{\propto A}=0.00}^{stDev_{\propto A}=0.24}\left(\varepsilon\left(stDev_{TA(0)},\ stDev_{\gamma A}, stDev_{\propto A}\right)\right)\right)\right)}{15625}$$

In the above equation $stDev_{TA(0)}$, $stDev_{\gamma A}$ and $stDev_{\alpha A}$ are the standard deviation values used to generate the agents' initial trust values, the agents' trust flexibility parameter, and agents' social influence parameter from a uniform normal distribution around the mean value of *0.50*. Here $\varepsilon(stDevT_{A(0)},\ stDev_{\gamma A},\ stDev_{\alpha A})$ is the error calculated for an experimental setup where $T_A(0)$, γ_A, and α_A were taken using $stDev_{TA(0)}$, $stDev_{\gamma A}$ and $stDev_{\alpha A}$ as standard deviation for a random number generator. Here it can be noted that to obtain the average, this summation is divided by *15625* which are the number of comparison models generated by all variations in $stDev_{TA(0)}$, $stDev_{\gamma A}$, and $stDev_{\alpha A}$, e.g. *25*25*25*.

In order to simulate realistic behavior of the trustee's experience *E* to the agents, *E* was also taken from a uniform normal distribution with mean value of *0.50* and experience's standard deviation $stDev_E$ from the interval *0.00 – 0.24*. These experience values were also taken exhaustively over $stDev_{TA(0)}$, $stDev_{\gamma A}$, and $stDev_{\alpha A}$. The algorithm for the simulation experiments is presented below; it compares the population-based trust model with the agent-based trust model exhaustively with all possible standard deviations of $stDev_E$, $stDev_{\gamma A}$, $stDev_{TA(0)}$ and $stDev_{\alpha A}$ varying in the interval *0.00-0.24* described as follows.

Algorithm S: Agent and population base model comparison

Agent [*A1, A2, …An*] of **ABM**, **Agent** *P* of **PBM**, **Trustee** S;
for all $stdDev_E$ from *0.00* to *0.24*
  for all $stdDev_{\gamma A}$ from *0.00* to *0.24*
    for all $stdDev_{TA(0)}$ from *0.00* to *0.24*
      for all $stdDev_{\alpha A}$ from *0.00* to *0.24*
        for all Agents A in **ABM**
          initialize $T_A(0)$ of A from $stdDev_{TA(0)}$
          initialize γ_A of *A* from $stdDev_{\gamma A}$
          initialize α_A of *A* from $stdDev_{\alpha A}$
        end for [all agents A]
        initialize $T_P(0)$, γ_P and α_P of *P* with average of $T_A(0)$, γ_A and α_A
        for all time points *t*
          trustee *S* gives experience E(t) from $stdDev_E$
          agent *P* receives $E_P{}^d(t)$ and calculates $E_P{}^i(t)$ where $E_P{}^d(t) = E(t)$
          agent *P* updates trust T_P(t) of *S*

for all agents A in **ABM**
A receives experience $E_A^d(t)$ where $E_A^d(t) = E(t)$
for all agents B in **ABM where A≠B**
A gets opinion $O_{AB}(t)$ from B and aggregate in $E_A^i(t)$
end for [all agents B]
A updates trust $T_A(t)$ on S
update $T_C(t)$ of S using trust $T_A(t)$ of A
end for [all agents A]
calculate error ε of models using $T_P(t)$ and $T_C(t)$
end for [all time points t]
end for [all agents $stdDev_{\alpha A}$]
end for [all $stdDev_{TA(0)}$]
calculate average models error ε_{avg} for all models($stdDev_{\gamma A}$, $stdDev_{TA(0),}$ $stdDev_{TA(0)}$)
end for [all $stdDev_{\gamma A}$]
calculate average experience level error ε_E for all experience sequences using ε_{avg}
end for [all $stdDev_E$]

3.2. Experimental Configurations

In Table 1 the experimental configurations used for the different simulations are summarized. All simulations were run for 500 time steps, and were performed for different values for the agents in the agent-based model to cover different types of populations. The parameter *SS* for the sample of simulation experiments is taken 25: each experiment is run 25 times after which an average is taken. This is meant to undo the randomization effects and to get the general average characteristics of the models. To obtain a wide variety of possible dynamics of the agent-based trust model the agents' initial trust, the agents' flexibility, agents' social influence and the experience with the trustee were taken exhaustively from a uniform normal distribution with various standard deviations.

Table 1. Experimental configurations.

Name	Symbol	Value
Total time steps	TT	*500*
Number of agents	N	*10, 20, 30, 40, 50*
Samples of simulation experiments	SS	*25*
Standard deviation and mean for direct experience	$stdDev_E$, $mean_E$	*0.00-0.24, 0.50*
Standard deviation and mean for rate of change	$stdDev_\gamma$, $mean_\gamma$	*0.00-0.24, 0.50*
Standard deviation and mean for initial trust	$stdDev_{T(0)}$,$mean_{T(0)}$	*0.00-0.24, 0.50*
Standard deviation and mean for social influence	$stdDev_\alpha$ $mean_\alpha$	*0.00-0.24, 0.50*

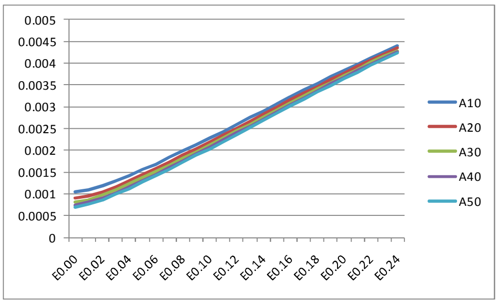
Given the above experimental configurations, time complexity for the simulation experiments for the algorithm S is $O(stdDev_E \,.\, stdDev_\gamma \,.\, stdDev_{TA(0)} \,.\, stdDev_\alpha \,.\, TT \,.\, N \,.\, SS)$. For $stdDev_E$ $stdDev_\gamma$, $stdDev_{TA(0)}$ and $stdDev_\alpha$ ranging from *0.00* to *0.24*, 500 time steps for simulation, 50 agents and 25 samples of simulation the approximate number for the instruction count becomes 1.22×10^{13}.

3.3. Simulation Results

The algorithm S specified in Section 3.1 was implemented in C++ to conduct the simulations experiments using the configuration as described in Table 1, and to compare the agent-based and population-based trust models. In this section some of the simulation results are discussed.

Variation in the experience value from the trustee

In this experiment an exhaustive simulation was performed where the trustee gives experience values from a uniform normal distribution around the mean value *0.50* with standard deviation $stdDev_E$ from the interval *0.00* to *0.24*. For each value of $stdDev_E$ the agents' initial trust, flexibility and social influence parameters were taken from a uniform normal distribution with mean value *0.50* and standard deviation varying from *0.00* to *0.24* (see algorithm S). To see the effect of the population size on this experiment, the experiment was executed for different numbers of agents varying from 10 to 50. Some of the results are shown in Fig. 2. In Fig. 2a) the horizontal axis represents the standard deviation in the experience values *E* given by the trustee, varying from *0.00* to *0.24* and the vertical axis shows the average experience level error ε_E of all models with standard deviations of the agent attributes $T_A(0)$, γ_A and α_A in the agent-based model, varying from *0.00* to *0.24*. Here it can be seen that upon an increase in standard deviation of experience value given by the trustee, the average error between the agent-based and population-based model increases for all population sizes (from about *0.001* to about *0.004*). This error values is lower for higher numbers of agents which shows that the population-based model is a much better approximation of the agent-based based model for higher number of agents. In Fig. 2b) the horizontal axis shows the number of agents in the agent-based model while the vertical axis represents the average of the experience level error ε_E for all models, where the trustee gives experience values with standard deviation $stdDev_E$ (varying from *0.00* to *0.24*), and the agents in the agent-based model have attributes $T_A(0)$, γ_A and α_A with standard deviations $stdDev_{\gamma A}$, $stdDev_{TA(0)}$, and $stdDev_\alpha$ (varying from *0.00* to *0.24*). Here it can also be observed that the population-based trust model provides a (slightly) more accurate approximation of the agent-based model, when having larger numbers of agents (from about *0.0026* to about *0.0024*).



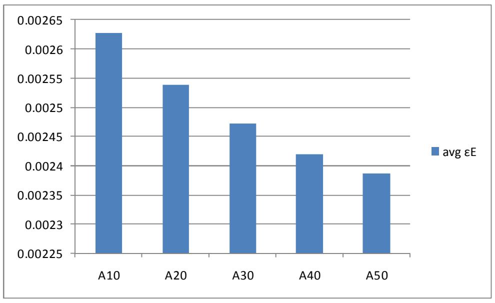


Figure 2. a) Difference between agent-based and population-based trust models upon variation in experience values
b) Average difference (error) between the agent-based and population-based trust models for all possible standard deviations of $stdDev_\gamma$, $stdDev_{TA(0)}$, $stdDev_\alpha$ and $stdDev_E$

In all these experiments the maximum root mean squared error between agent-based and population-based trust model does not exceed *0.027267*, which means that this population-based trust model is a quite accurate approximation of the agent-based model.

Exhaustive mirroring of agent-based model into population-based model

In the previous experiment the attribute values of the population-level model were simply taken as an average of the attribute values of all agents in the agent-level model. However, it cannot be claimed at forehand that this mechanism of abstracting the agent-level model is the most accurate aggregation technique. In order to see whether there is any other instance of the population-level model that can approximate the agent-level models better then the one based on aggregating by averaging, one has to exhaustively simulate all instances of the population-based model against all instances of the agent-based model. In this experiment such an exhaustive simulation was performed, applying a method named as *exhaustive mirroring of models*, adopted from [6]. In this method of mirroring of models the target model is exhaustively (for different parameter settings) simulated to realize a specific trace of the source model for a given set of parameters of source model. The instance of the target model for specific values of the parameters that generate a minimal error is considered as the best realization of the target model to approximate the source model. As stated in [6] this process gives some measure of similarity of the target model against the source model. However, this method of exhaustive mirroring is computationally very expensive. So, for practical reasons in this experiment the population-based model (target) is exhaustively simulated with only one of the three population level parameters, namely the flexibility γ_P of the population-level trust. The other two parameters the population-level (initial trust $T_P(0)$ and social influence α_P) were taken as the average of their counterparts in the agent-level model. Some of the results of this experiment are shown in Fig. 3; In Fig. 3a) the horizontal axis represents the exhaustive values for the population-level flexibility parameter γ_P and the vertical axis shows the average experience level error ε_E of all agent-based models with standard deviations of the attributes $T_A(0)$, γ_A, α_A and trustee experience $E^d(t)$ varying from *0.00* to *0.24* with mean value *0.5*. Here it can be seen that for lower values of γ_P the average error is much higher and it starts to reduce when γ_P approaches to *0.5* and values of γ_P above *0.5* this error starts to increase. Hence *0.50* is the most accurate representation of γ_P for all agent base models. Further in Fig. 3b) same graph is shown in a zoomed-in fashion to show the effect of population size on error value. Here it is seen that larger populations showed lower error than smaller populations.

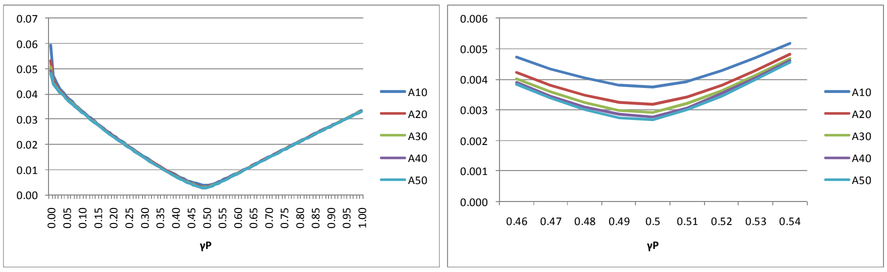


Figure 3. a) Difference between agent-based and population-based trust models upon change in population level flexibility parameter γ_P, b) Zoomed-in version of Fig. 3a)

Comparison for larger populations with numbers up to *500* agents

Based on observation from the experiments described above some support was obtained that the value *0.5* for the population-level flexibility parameter γ_P is the most accurate representation of the agent-based model. To get a better impression for the limit value of the error for larger populations, in the next experiment the agent-based model were simulated for larger populations up to *500* agents in size and compared to the population-based model with flexibility parameter $\gamma_P = 0.5$. In this experiment the population was varied from *200* to *500* agents with an increment of *10* agents per population size. Experimental configurations in this experiment were taken from Table 1. Results are shown in Fig. 4; In Fig. 4a) the horizontal axis represents the different population sizes varying from 200 to 500 agents and the vertical axis shows the average difference between agent and population level models. Here it can be seen that on an increase in number of agents in population base model difference between models decreases from about *0.00229* (for *200* agents) to about *0.00218* (for *500* agents). It has been analysed in how far the approximation of the limit value for the error for larger populations is exponential and how the limit value can be estimated from the obtained trend. To this end Fig. 4b) depicts for a certain value of e (an assumed limit value) the graph of the logarithm of the distance of the error to e, expressed as $ln(error - e)$. This graph (in blue) is compared to a straight line (in red). It turns out that in 6 decimals the straight line is approximated best for limit value $e = 0.002145$, and the approximation of this limit value for e goes exponentially according to an average (geometric mean) factor *0.947149* per increase of *10* agents. In summary, given that the error found for $N = 200$ is *0.002288*, based on this extrapolation method the difference between the agent-based and population-based model for larger population sizes $N \geq 200$ can be estimated as

$$
\begin{aligned}
est\_error(N) &= 0.002145 + (0.002288-0.002145)*0.947149^{N-200} \\
&= 0.002145 + 0.000143*0.947149^{N-200}
\end{aligned}
$$

This estimation predicts that always an error of at least *0.002145* is to be expected; this actually is quite low, but it will not become still lower in the limit for very large N. It turns out that the difference between actual error and estimated error using the above formula for all N between *200* and *500* is less than 2.10^{-6}, with an average of 7.10^{-7}. Note that by having this estimation of the error, it can also be used to correct the population-based model for it, thus in a cheap manner approximating the agent-based model by an accuracy around 10^{-6}.

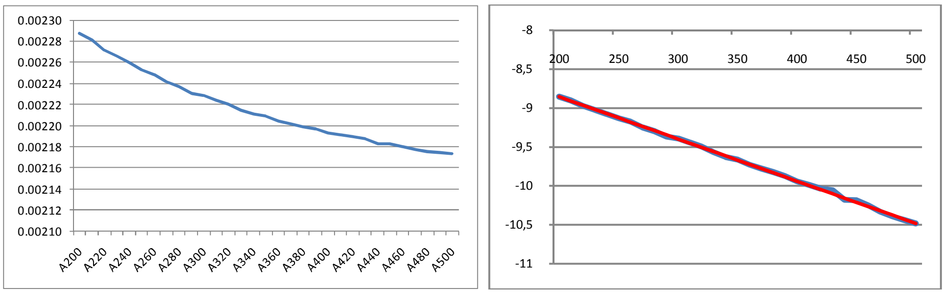


Figure 4. a) Difference between agent-based and population-based trust models upon change in population size on level flexibility parameter $\gamma_P = 0.5$
b) Graph of *ln(error – e)* compared to a straight line for *e = 0.002145*

4 Mathematical Analysis of Equilibria of the Two Models

The agent-based and population-based models can also be analysed mathematically by determining equilibria. These are values for the variables upon which no change occcurs anymore. For equilibria also the externally given experience values have to be constant; instead of these values for E also the expectation value for them can be taken. For the population-level model, assuming flexibility $\gamma_P > 0$ an equilibrium has to satisfy

$$T_P(t) = E_P(t)$$

with

$$E_P(t) = \alpha_P \, T_P \ (t) + (1 - \alpha_P) E_P^d(t)$$

Leaving out t, and taking $E = E_P^d$, this provides the following equation in T_P

$$T_P = \alpha_P \, T_P + (1\text{-}\alpha_P)E$$

Thus (assuming $\alpha_P \neq 1$) an equilibrium $T_P = E$ is obtained.

In a similar manner for the agent-based model equilibria can be determined. Again, assuming flexibility $\gamma_A > 0$ an equilibrium has to satisfy for each agent A

$$T_A(t) = E_A(t)$$

this time with

$$E_A(t) = \alpha_A \, E_A^i(t) + (1 - \alpha_A) E_A^d(t)$$

where

$E_A^i(t) = \Sigma_{B \neq A}\ T_B(t)/(N\text{-}1)$

This provides N equations

$$T_A(t) = \alpha_A\ \Sigma_{B \neq A}\ T_B(t)/(N\text{-}1) + (1\text{-}\alpha_A)E$$

By aggregating these equations, and leaving out t, the relation to collective trust can be found:

$$\begin{aligned}
\Sigma_A\ T_A/N &= \Sigma_A\ [\alpha_A\ \Sigma_{B \neq A}\ T_B/(N\text{-}1) + (1\text{-}\alpha_A)E\ ]/N \\
T_C &= \Sigma_A\ \alpha_A\ \Sigma_{B \neq A}\ T_B/(N\text{-}1)N + \Sigma_A\ (1\text{-}\alpha_A)E\ /N \\
&= \Sigma_A\ \alpha_A\ [\Sigma_B\ T_B - T_A]/(N\text{-}1)N + (1\text{-}\Sigma_A\ \alpha_A/N)\ E \\
&= [\ \Sigma_A\ \alpha_A\ \Sigma_B\ T_B - \Sigma_A\ \alpha_A\ T_A]/(N\text{-}1)N + (1\text{-}\Sigma_A\ \alpha_A/N)\ E \\
&= [\ \Sigma_A\ \alpha_A\ T_C\ /(N\text{-}1) - \Sigma_A\ \alpha_A\ T_A\ /(N\text{-}1)N] + (1\text{-}\Sigma_A\ \alpha_A/N)\ E \\
&= [(\Sigma_A\ \alpha_A\ /N)\ T_C\ N\ /(N\text{-}1) - \Sigma_A\ \alpha_A\ T_A\ /(N\text{-}1)N] + (1\text{-}\Sigma_A\ \alpha_A/N)\ E\ T_C \\
&= [(\Sigma_A\ \alpha_A\ /N)\ T_C + (\Sigma_A\ \alpha_A\ /N)\ T_C/(N\text{-}1)\ - \Sigma_A\ \alpha_A\ T_A\ /(N\text{-}1)N] + (1\text{-}\Sigma_A\ \alpha_A/N)\ E \\
&= (\Sigma_A\ \alpha_A\ /N)\ T_C + (1\text{-}\Sigma_A\ \alpha_A/N)\ E + [(\Sigma_A\ \alpha_A\ /N)\ T_C/(N\text{-}1) - \Sigma_A\ \alpha_A\ T_A\ /(N\text{-}1)N] \\
&= (\Sigma_A\ \alpha_A\ /N)\ T_C + (1\text{-}\Sigma_A\ \alpha_A/N)\ E + [(\Sigma_A\ \alpha_A T_C - \Sigma_A\ \alpha_A\ T_A]\ /(N\text{-}1)N \\
&= (\Sigma_A\ \alpha_A\ /N)\ T_C + (1\text{-}\Sigma_A\ \alpha_A/N)\ E + \ \Sigma_A\ \alpha_A[T_C - T_A]\ /(N\text{-}1)N
\end{aligned}$$

So, taking $\alpha_C\ = \Sigma_A\ \alpha_A\ /N$ the folowing equilibrium equation is obtained:

$$\begin{aligned}
(1\text{-}\alpha_C)\ T_C &= (1\text{-}\alpha_C)\ \ E + \Sigma_A\ \alpha_A[T_C - T_A]\ /(N\text{-}1)N \\
T_C &= \ E + \Sigma_A\ \alpha_A[T_C - T_A]\ /(N\text{-}1)N(1\text{-}\alpha_C)
\end{aligned}$$

Therefore in general the difference between the equilibrium values for T_C (aggregated agent-based model) and T_P (population-based model) can be estimated as

$$T_C - T_P = \ T_C - E\ =\ \Sigma_A\ \alpha_A[T_C - T_A]\ /(N\text{-}1)N(1\text{-}\alpha_C)$$

As T_C and T_A are both between 0 and 1, the absolute value of the expression in $T_C - T_A$ can be bounded as follows

$$|\Sigma_A\ \alpha_A[T_C - T_A]\ /(N\text{-}1)N(1\text{-}\alpha_C)| \leq \Sigma_A\ \alpha_A\ /(N\text{-}1)N(1\text{-}\alpha_C)| = \alpha_C/(N\text{-}1)(1\text{-}\alpha_C)$$

Therefore the following bound for the difference in equilibrium values is found:

$$|T_C - T_P|\ \leq\ \alpha_C\ /(N\text{-}1)(1\text{-}\alpha_C)$$

This goes to 0 for large N, which would provide the value $T_C = E = T_P$. For $\alpha_C = 0.5$, and $N = 200$, this bound is about 0.005, for $N = 500$, it is about 0.002. These deviations are in the same order of magnitude as the ones found in the simulations. Note that the expression in $T_C - T_A$ also depends on the variation in the population. When all agents have equal characteristics $\alpha_A = \alpha$ it is 0, so that $T_C = E = T_P$.

$$T_C - T_P = \alpha \Sigma_A [T_C - T_A] /(N-1)N(1-\alpha_C)$$
$$= \alpha [\Sigma_A T_C / N - \Sigma_A T_A/ N] /(N-1) (1-\alpha_C)$$
$$= \alpha [T_C - T_C] /(N-1)(1-\alpha_C)$$
$$= 0$$

So also in the case of equal parameter values for α_A it holds $T_C = E = T_P$; note that this is independent of the variation for the other parameters.

5 Conclusion

This paper addressed an exploration of the differences between agent-based and population-based models for trust dynamics, based on both a large variety of simulation experiments and a mathematical analysis of the equilibria of the two types of models. By both types of exploration it was shown that the differences between the two types of model are quite small, in general below *1%*, and become less for larger numbers of agents. An implication of this is that when for a certain application such an accuracy is acceptable, instead of the computationally more expensive agent-based modelling approach (complexity $O(N^2\tau)$ with N the number of agents and τ the number of time steps), as an approximation also the population-based approach can be used (complexity $O(\tau)$).

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