Agent-Based and Population-Based Modeling of Trust Dynamics[1]

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Abstract: Trust is usually viewed at an individual level in the sense of an agent having trust in a certain trustee. It can also be considered at a population level, in the sense of how much trust for a certain trustee exists in a given population or group of agents. The dynamics of trust states over time can be modelled per individual in an agent-based manner. These individual trust states can be aggregated to obtain the trust state of the population. However, in an alternative way trust dynamics can be modelled from a population perspective as well. Such a population-level model is much more efficient computationally. In this paper both ways of modelling are investigated and it is analyzed how close they can approximate each other. This is done both by simulation experiments and by mathematical analysis. It is shown that the approximation can be reasonably accurate, and for larger numbers of agents even quite accurate.

1 Introduction

Trust is a concept that is usually considered as a means to aggregate experiences with particular issues (trustees), such as other agents or services; e.g., [23, 25, 27]. A trust value can be taken into account in decisions, for example, when deciding on cooperation with other agents. A variety of computational models for trust dynamics has been developed; see e.g. [8, 15, 16, 21, 22]. Trust can be considered at the individual level, but also at a collective level for a population as a whole, for example when decisions are made by voting. When a population of agents is considered, the dynamics of trust in a certain trustee can be modelled from two perspectives: from the agent-based per-

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spective and from the population-based perspective. From the agent-based perspective each agent has its own characteristics and maintains its own trust level over time. From the population-based perspective one trust level for the whole is maintained over time, depending on characteristics of the population. For both cases dynamical models can be used to determine the trust levels over time. For the agent-based perspective, each agent has its own dynamical model (for example, expressed as a system of N sets of differential equations with N the number of agents, one for each agent describing temporal relations for the agent's states), whereas for the population-level one model (for example, expressed as one set of differential equations describing temporal relations for population states) can be used. From the agent-based model, by aggregation of the individual agent trust states, a collective trust level for the population as a whole can be determined, for example, by taking the average of the trust levels over all agents. Note that this distinction is different from the distinction agent-based vs equation-based made in [20]. In their case the criterion is more on the form in which the model is specified:

> 'The differences are in the form of the model and how it is executed. In agent-based modeling (ABM), the model consists of a set of agents that encapsulate the behaviors of the various individuals that make up the system, and execution consists of emulating these behaviors. In equation-based modeling (EBM), the model is a set of equations, and execution consists of evaluating them.' [20, p. 10]

In the distinction used in the current paper the criterion is not on the form in which the model is specified (both can be specified by equations) but on whether the concepts used in the model refer to states of an individual agent, or to states of the population as a whole. For example, if an epidemics model uses three concepts or variables that indicate the total numbers of susceptible persons, infectuous persons and recovered persons, then these concepts or variables refer to population states, independent of whether dynamical relations between them are specified in logical, numerical or other formats. The model then is a population-level model. If instead in a model there are concepts or variables for each individual agent, for example that agent A is susceptible, then the model is agent-based, again independent of the format in which relations between these concepts are specified.

Within application disciplines such as Biology, Economics, and Medicine, the classical dynamical modeling approaches for simulation of processes in which larger numbers of agents are involved are population-based: a population is represented by a numerical variable indicating its number or density (within a given area) at a certain time point. The dynamical model takes the form of a system of difference or differential equations for the dynamics of these variables. Well known classical examples of such population-based models are systems of difference or differential equations for predator-prey dynamics [e.g., 4, 18, 19, 26] and the dynamics of epidemics [e.g., 1, 4, 7, 17]. Historically such population level models are specified and studied by simulation and by numerical-mathematical techniques.

The agent-based perspective often takes as a presupposition that simulations based on individual agents are a more faithful way of modelling, and thus will provide better results [e.g., 2, 6, 23]. Although for larger numbers of agents agent-based dynamical modelling approaches are more expensive computationally than population-based modelling approaches, such a presupposition provides a justification of preferring their use over population-based modelling approaches: agent-based approaches with larger numbers of agents are justified because the results are expected to be more realistic than the results of population-based simulation. This may not work the same for smaller and for larger numbers of agents. For larger numbers of agents, by some form of averaging, population-based simulations might be an adaequate approximation of agent-based simulations . If so, for applications with larger numbers of agents in which the agent-based model is difficult to use due to its computational complexity, this implies that population-based simulation would be a good choice. However, there are also many cases that the agent-based approach has a manageable complexity, and then the choice may be made to use an agent-based approach.

In this paper above assumption is explored in a detailed manner for a population of agents that receive direct experiences from a trustee and also get communicated information from other agents about their trust in this trustee. On one hand an analysis is performed that makes use of a variety of simulation experiments for different population sizes and different distributions of characteristics while on the other a mathematical analysis of equilibria of both types of models is used to find out differences between the two. Roughly spoken, the outcome of both type of investigations are that in general the differences are not substantial, and that they are smaller the larger the number of agents is. In Section 2 the two types of trust models used are introduced which incorporate direct and indirect experiences. In Section 3 the simulation experiments are described. Section 4 presents the mathematical analysis of equilibria. Section 5 concludes the paper.

2 Modelling Trust Dynamics from Two Perspectives

In this section trust models for both perspectives are introduced. The basic underlying trust dynamics model adopted in this paper depends on receiving experiences *E(t)* over time as follows:

$$T(t + \Delta t) = T(t) + \gamma * \left(E(t) - T(t)\right) * \Delta t \qquad (1)$$

Here *T(t)* and *E(t)* are the trust level for a trustee and the experience level given by the trustee at time point *t*. Furthermore, γ is a personal characteristic for flexibility: the rate of change of trust upon receiving an experience *E(t)*. The values of *T(t)*, *E(t)* and γ are in the interval [*0*, *1*] where *0* value for *E(t)* and *T(t)* means absolute bad experience and no trust at all while *1* for *E(t)* and *T(t)* means absolute positive experience and absolute trust respectively. In differential form change of trust over time can be expressed by:

$$\frac{dT}{dt} = \gamma * (E - T) \qquad (2)$$

This basic model is based on the experienced-based trust model described in [14], and applied in [23, 24, 26]. In an agent's received experience, experiences are taken to be of two forms: direct experiences, for example, by observation, and indirect experiences obtained from communication. Here the model presented in (2) is applied in both cases where trust updates is only based on direct experience received from the trustee and where it depends on both direct and indirect experiences received from trustee and other agents. Incorporating this, the basic model can be applied to each single agent within the population (agent-based perspective), or to the population as a whole (population-based perspective), as discussed below.

2.1 The Agent-Based Trust Model using Direct Experience

In this section the agent-based trust model is described for a trustee. Each of the agents updates its trust on a given trustee upon receiving an experience from this trustee. The trust model described above is taken and indexed for each agent *A* in the group:

$$T_A(t + \Delta t) = T_A(t) + \gamma_A * \left(E_A(t) - T_A(t)\right) * \Delta t \quad (3)$$

Note that each agent can have its personal flexibility characteristic γ_A in interval *[0, 1]*. It is assumed that these values have some distribution over the population. Moreover each agent may have its own experiences $E_A(t)$; however, in this paper these are assumed the same for all agents.

Using the agent-based model a collective trust value $T_C(t)$ for the population of N number of agents as a whole can be obtained by aggregation of the trust values over all agents (taking the average):

$$T_C(t) = \frac{1}{N} \sum_{A=1}^{N} T_A(t) \quad (4)$$

2.2 The Population-Based Trust Model using Direct Experience

In this section the population-based model of trust used is described. Besides the experience given by the trustee to a population *P*, the dynamics of population level trust are influenced by the characteristics of the population [12]. Here *P* is defined as a population of *N* agents receiving same experiences from a trustee and/or communicating with each other. A population level model has to aggregate the diversity of the population. The proposed population-based model of trust carries all dynamics as described in base model as follows:

$$T_P(t + \Delta t) = T_P(t) + \gamma_P * \left(E_P(t) - T_P(t)\right) * \Delta t \quad (5)$$

Here $T_P(t)$ is the collective trust of whole population on a given trustee at time point *t*, and the population-level flexibility characteristic γ_P is an aggregate value for the individual flexibility characteristics γ_A for all agents present in *P* (e.g., the average of the γ_A for $A \in P$). Conceptually this can be interpreted as if the population as a whole is represented as one agent and receives experiences from the trustee and updates its trust on the trustee according to equation 5. Note again that the experience levels $E_P(t)$

and $E_A(t)$ for the population P and all individual agents A are assumed to be the same. Therefore the index for $E(t)$ can be left out.

2.3 An Agent-Based Trust Model Incorporating Indirect Experience

In the agent-based trust model for a trustee described here, each of the agents updates its trust on a given trustee based on receiving an experience for this trustee and combines a direct experience and an opinion received by the peers about the trustee (indirect experience). Direct and indirect experiences at each time point are aggregated using agents' personality characteristic called social influence denoted by α_A as follows:

$$E_A(t) = \alpha_A * E_A^i(t) + (1 - \alpha_A) * E_A^d(t) \tag{6}$$

Here $E_A(t)$, $E_A^d(t)$ and $E_A^i(t)$ are the aggregated experience, the direct experience received from the trustee and the indirect experience received by the agent A as the opinions of its peers about trustee at time t respectively. The social influence (α_A) can have value in the interval $[0, 1]$ where 0 value for α_A means agent will only consider direct experiences while 1 means only indirect experiences would be counted in agent's aggregated experiences. The indirect experience $E_A^i(t)$ received by the agent A as the opinions of its peers about trustee at time point t is taken the average of the opinions given by all the peers at time point t:

$$E_A^i(t) = \frac{\sum_{B=1\ and\ B \neq A}^{N} O_B(t)}{(N-1)} \tag{7}$$

Here $O_B(t)$ is the opinion received by the agent A from an agent B about the trustee at time point t and N is the total number of agents in the population. The opinion given by the agent B to the agent A at time t is taken as the value of the trust of B on trustee at time t, e.g. $O_B(t) = T_B(t)$, hence equation 7 will become as follows:

$$E_A^i(t) = \frac{\sum_{B=1\ and\ B \neq A}^{N} T_B(t)}{(N-1)} \tag{8}$$

The aggregated experience received by agent A at time point t as expressed in equation (6) is used to update current trust level of the agent A at trustee using trust model presented in equation 3 as follows:

$$T_A(t + \Delta t) = T_A(t) + \gamma_A * \left(E_A(t) - T_A(t)\right) * \Delta t \tag{9}$$

Here the basic trust model is indexed for each agent A in the group. Note that each agent can have its personal flexibility characteristic γ_A. It is assumed that these values have some distribution over the population. Based on this agent-based model a collective trust value $T_C(t)$ for the population as a whole can be obtained by aggregation of the trust values over all agents (taking the average):

$$T_C(t) = \frac{\sum_{A=1}^{N} T_A(t)}{N} \tag{10}$$

2.4 A Population-Based Trust Model Incorporating Indirect Experience

To apply the basic trust model to obtain a population-based model of trust, its ingredients have to be considered for the population *P* as a whole, for example, the (direct and indirect) experience given by the trustee to a population *P*, and the characteristics γ_P of the population [9, 12] this is done as follows:

$$T_P(t + \Delta t) = T_P(t) + \gamma_P * \left(E_P(t) - T_P(t)\right) * \Delta t \quad (11)$$

Here $T_P(t)$ is the trust of population *P* on a given trustee at time point *t*, and the population-level flexibility characteristic γ_P is taken as an aggregate value for the individual flexibility characteristics γ_A for all agents *A* in *P* (e.g., the average of the γ_A for $A \in P$). This can be interpreted as if the population as a whole is represented as one agent who receives experiences from the trustee and updates its trust on the trustee using the basic model. The experience at population level $E_P(t)$ at time point *t* for the population *P* is defined as the combination of the direct and the indirect experience at population level as follows:

$$E_P(t) = \alpha_P * E_P^i(t) + (1 - \alpha_P) * E_P^d(t) \quad (12)$$

In equation (12), $E_P^i(t)$ and $E_P^d(t)$ are the indirect and direct experience at the population level. Moreover, α_P is the population-level social influence characteristic. Here also α_P is taken as an aggregate value for the individual social influence characteristics α_A for all agents present in *P* (e.g., the average of the α_A for $A \in P$). At the population level the indirect experience $E_P^i(t)$ obtained from communication by the other agents of their trust is taken as the population level trust value at time point *t* as follows:

$$E_P^i(t) = T_P(t) \quad (13)$$

2.5 Computational Complexity Estimation

The computational complexity of the agent-based trust model differs from that of the population-based trust model. Computational complexity of the agent-based trust model depends on the number of agents while population-based model is independent of this. These complexities can be estimated as follows, if τ is the total number of time steps, and *N* the number of agents in the population, the time complexities of the agent-based trust model based on direct experience, agent-based trust model incorporating indirect experience and population-based trust models are $O(N\tau)$, $O(N^2\tau)$ and $O(\tau)$ respectively. This indicates that for higher numbers of agents in a population the agent-based model is computationally much more expensive than the population-based model.

3 Simulating and Comparing the Two Approaches

A number of simulation experiments have been conducted to compare the agent-based and population-based trust models as described in the previous sections. This section presents the experimental setup and results from these experiments.

3.1 The Experimental Setup for Trust models incorporating direct experiences

Several simulation experiments have been performed to analyze and compare the behaviour of the two approaches. For these simulation experiments a system was created as described in Fig. 1. Here it is assumed that a trustee *S* gives similar experiences *E* to both models at each time point. In the population-based trust model this experience *E* is used to update the population-level trust of *S* according to the equations presented in the last section, while in the agent-based trust model this experience is received by every agent in the system and each agent updates its trust on the trustee, which by aggregation leads to update of the collective trust of the trustee.

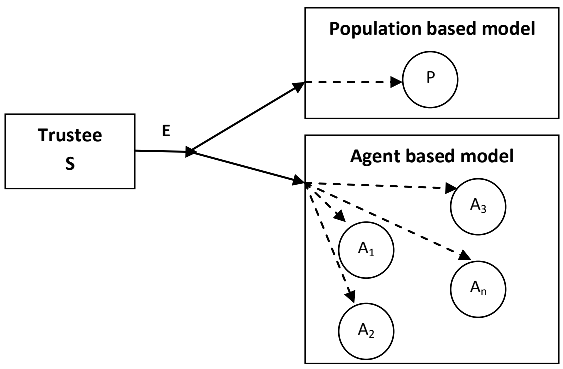


Fig. 1. Agent and Population based trust model

In Fig. 1, agent P carries the population-based trust model while agents A_1, A_2, A_3 … A_n carry the agent-based trust model as described in the previous Sections 2.1 and 2.2. Every agent in the system is assigned an initial trust value $T_A(0)$ and the value for the agent's flexibility parameter γ_A at the start of the simulation experiment. The value $T_P(0)$ for the initial population-level trust and the population-level flexibility parameter γ_P for the agent *P* of the population-based trust model are taken as the average of the corresponding attributes of all the agents in the community:

$$T_P(0) = \sum_{A=1}^{A=N} T_A(0)/N \tag{14}$$

and

$$\gamma_P = \sum_{A=1}^{A=N} \gamma_A/N \tag{15}$$

Here N is the total number of agents in the community. The collective trust of the agent-based trust model at any time point t is represented as the average of the trust values of all the agents in the community:

$$T_C(t) = \sum_{A=1}^{A=N} T_A(t)/N \tag{16}$$

As a measure of dissimilarity for the comparison of the models their root mean square error is measured between the collective agent-level trust and population-level trust at each time point t as follows:

$$\varepsilon = \sqrt{\sum_{t=1}^{t=M} (T_P(t) - T_C(t))^2 / M} \tag{17}$$

In equation (17), $T_P(t)$ and $T_C(t)$ are the population-level trust and the (aggregated) collective agent-level trust of trustee calculated by the population-based and agent-based model at time point t respectively and M is the total time steps in the simulation.

To produce realistic simulations, the values for $T_A(0)$ and γ_A of all the agents in the agent-based trust model were taken from a discretised uniform normal distribution with mean value *0.50* and standard deviation varying from *0.00* to *0.24*. In these experiments all agent-based models were simulated exhaustively to see their average behaviour against the population-based model. Here the exhaustive simulation means that all possible combinations of standard deviations for $T_A(0)$ and γ_A from the interval *0.00-0.24* were used in the simulations and their respective errors were measured. An average error ε_{avg} of the models was calculated, which is the average of all root mean squared errors calculated with all combinations of $T_A(0)$ and γ_A as follows:

$$\varepsilon_{avg} = \frac{\sum_{stDev_{TA(0)}=0.00}^{stDev_{TA(0)}=0.24} \left(\sum_{stDev_{\gamma A}=0.00}^{stDev_{\gamma A}=0.24} \left(\varepsilon_{\left(stDev_{TA(0)}, stDev_{\gamma A}\right)} \right) \right)}{625} \tag{18}$$

In equation (18), $stDev_{TA(0)}$ and $stDev_{\gamma A}$ are the standard deviation values used to generate the agents' initial trust values and the agents' trust flexibility parameters from a discretised uniform normal distribution around the mean value of *0.50*. Here $\varepsilon_{(stDevTA(0),\ stDev\gamma A)}$ is the error calculated for an experimental setup where $T_A(0)$ and γ_A were taken using $stDev_{TA(0)}$ and $stDev_{\gamma A}$ as standard deviation for a random number generator. Here it can be noted that to obtain the average, this summation is divided by *625* which is the number of comparison models generated by all variations in $stDev_{TA(0)}$ and $stDev_{\gamma A}$, e.g. *25*25*.

In order to simulate realistic behaviour of the trustee's experience E to the agents, E was also taken from a discretised uniform normal distribution with mean value of *0.50* and experience's standard deviation $stDev_E$ from the interval *0.00 – 0.24*. These experience values were also taken exhaustively over $stDev_{TA(0)}$ and $stDev_{\gamma A}$. The algorithm for the simulation experiments is presented below as ALGORITHM S1; it compares the population-based trust model with the agent-based trust model exhaustively with all possible standard deviations of $stDev_E$, $stDev_{\gamma A}$, $stDev_{TA(0)}$ varying in the interval *0.00-0.24* described as follows:

ALGORITHM S1: ABM AND PBM COMPARISON INCORPORATING DIRECT EXPERIENCES

```
Agent [A1, A2,…An] of ABM, Agent P of PBM, Trustee S;
for all stdDev_E from 0.00 to 0.24
  for all stdDev_γA from 0.00 to 0.24
    for all stdDev_TA(0) from 0.00 to 0.24
      for all Agents A in ABM
      initialize T_A(0) of A from stdDev_TA(0)
      initialize γ_A of A from stdDev_γA
      end for
    initialize T_P(0) of P with average of T_A(0) for all
    agents A
    initialize γ_P of P with an average of γ_A for all
    agents A
    for all time points t
    trustee S gives an experience E from stdDev_E
    agent P receives E
    agent P updates trust T_P of S
      for all agents A in ABM
      A receives experience E
      A updates trust T_A on S
      update T_C of S using trust T_A of A
      end for
    calculate error ε of models using T_P and T_C
    end for
  end for
  calculate average models error ε_avg for all possible
  models with stdDev_γA and stdDev_TA(0)
  end for
calculate average experience level error ε_E for all
experience sequences using ε_avg
end for
```

3.2 The Experimental Setup for Trust models incorporating indirect experiences

For the simulation experiments of trust models incorporating indirect experiences a setup was used as shown in Fig. 2. Here a trustee *S* is assumed to give similar direct experiences $E^d(t)$ to both models at each time point *t*. In the population-based trust model this direct experience $E^d(t)$ is used together with the indirect experience $E_P^i(t)$ to update the population-level trust of *S* according to the equations presented in Section 2.4. In the agent-based trust model this experience is received by every agent in the system and each agent updates its trust on the trustee using direct experience $E^d(t)$ and indirect experience $E_A^i(t)$ received as opinion of the other agents, as shown in

Section 2.3. By aggregation the individual trust levels can be used to obtain a collective trust of the trustee.

In Fig. 2 *P* carries the population-based trust model while the agents A_1, A_2, A_3 ... A_n, carry the agent-based trust model as described in the previous Sections 2.3 and 2.4. Every agent in the system is assigned an initial trust value $T_A(0)$, a value for the agent's flexibility γ_A, and for the social influence parameter α_A at the start of the simulation experiment. The value $T_P(0)$ for the initial population-level trust, the population-level flexibility parameter γ_P and the social influence α_A parameter for the population-based trust model are taken as the average of the corresponding attributes of all the agents in the community:

$$T_P(0) = \frac{\sum_{A=1}^{N} T_A(0)}{N} \tag{19}$$

$$\gamma_P = \frac{\sum_{A=1}^{N} \gamma_A}{N} \tag{20}$$

$$\alpha_P = \frac{\sum_{A=1}^{N} \alpha_A}{N} \tag{21}$$

Here *N* is the total number of agents in the community. The collective trust of the agent-based trust model at any time point *t* is represented as the average of the trust values of all the agents in the community: $T_C(0) = \frac{\sum_{A=1}^{N} T_A(0)}{N}$.

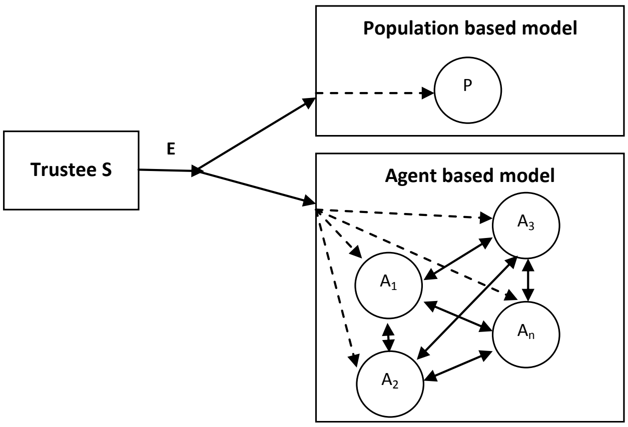


Fig. 2. Agent-based and population-based trust model incorporating communication

As a measure of dissimilarity for the comparison of the models their root mean square error is measured between the collective agent-level trust and population-level trust at each time point *t* as follows

$$\varepsilon = \sqrt{\sum_{t=1}^{t=M} (T_P(t) - T_C(t))^2 / M} \tag{22}$$

In equation (22), $T_P(t)$ and $T_C(t)$ are the population-level trust and the (aggregated) collective agent-level trust of the trustee calculated by the population-based and agent-based model at time point t respectively and M is the total time steps in the simulation.

To produce realistic simulations, the values for $T_A(0)$, γ_A and α_A of all the agents in the agent-based trust model were taken from a discretised uniform normal distribution with mean value *0.50* and standard deviation varying from *0.00* to *0.24*. In these experiments all agent-based models were simulated exhaustively to see their average behavior against the population-based model. Here exhaustive simulation means that all possible combinations of standard deviations for $T_A(0)$, γ_A and α_A from the interval *0.00-0.24* were used in the simulations and their respective errors were measured against respective population level model. An average error ε_{avg} of the models was calculated, which is the average of all root mean squared errors calculated with all combinations of $T_A(0)$, γ_A and α_A as follows.

$$\varepsilon_{avg} = \frac{\sum_{stDev_{TA(0)}=0.00}^{stDev_{TA(0)}=0.24}\left(\sum_{stDev_{\gamma A}=0.00}^{stDev_{\gamma A}=0.24}\left(\sum_{stDev_{\propto A}=0.00}^{stDev_{\propto A}=0.24}\left(\varepsilon\left(stDev_{TA(0)}, stDev_{\gamma A}, stDev_{\propto A}\right)\right)\right)\right)}{15625} \qquad (23)$$

In equation (23), $stDev_{TA(0)}$, $stDev_{\gamma A}$ and $stDev_{\alpha A}$ are the standard deviation values used to generate the agents' initial trust values, the agents' trust flexibility parameter, and agents' social influence parameter from a discretised uniform normal distribution around the mean value of *0.50*. Here $\varepsilon(stDevT_{A(0)}, stDev_{\gamma A}, stDev_{\alpha A})$ is the error calculated for an experimental setup where $T_A(0)$, γ_A, and α_A were taken using $stDev_{TA(0)}$, $stDev_{\gamma A}$ and $stDev_{\alpha A}$ as standard deviation for a random number generator. Here it can be noted that to obtain the average, this summation is divided by *15625* which are the number of comparison models generated by all variations in $stDev_{TA(0)}$, $stDev_{\gamma A}$, and $stDev_{\alpha A}$, e.g. *25*25*25*.

In order to simulate realistic behavior of the trustee's experience E to the agents, E was also taken from a discretised uniform normal distribution with mean value of *0.50* and experience's standard deviation $stDev_E$ from the interval *0.00 – 0.24*. These experience values were also taken exhaustively over $stDev_{TA(0)}$, $stDev_{\gamma A}$, and $stDev_{\alpha A}$. The algorithm for the simulation experiments is presented below as ALGORITHM S2; it compares the population-based trust model with the agent-based trust model exhaustively with all possible standard deviations of $stDev_E$, $stDev_{\gamma A}$, $stDev_{TA(0)}$ and $stDev_{\alpha A}$ varying in the interval *0.00-0.24* described as ALGORITHM S2.

ALGORITHM S2: ABM AND PBM COMPARISON INCORPORATING INDIRECT EXPERIENCES

```
Agent [A1, A2,…An] of ABM, Agent P of PBM, Trustee S;
for all stdDev_E from 0.00 to 0.24
  for all stdDev_γA from 0.00 to 0.24
    for all stdDev_TA(0) from 0.00 to 0.24
      for all stdDev_αA from 0.00 to 0.24
        for all Agents A in ABM
        initialize T_A(0) of A from stdDev_TA(0)
        initialize γ_A of A from stdDev_γA
        initialize α_A of A from stdDev_αA
        end for [all agents A]
```

```
      initialize T_P(0), γ_P and α_P of P with average of
      T_A(0), γ_A and α_A for all agents A
        for all time points t
        trustee S gives experience E(t) from stdDev_E
        agent P receives E_P^d(t) and calculates E_P^i(t)
        where E_P^d(t) = E(t)
        agent P updates trust T_P(t) of S
          for all agents A in ABM
          A receives experience E_A^d(t) where E_A^d(t)=E(t)
            for all agents B in ABM where A≠B
              A gets opinion O_AB(t) from B and aggre-
              gate in E_A^i(t)
            end for [all agents B]
          A updates trust T_A(t) on S
          update T_C(t) of S using trust T_A(t) of A
          end for [all agents A]
        calculate error ε of models using T_P(t) and
        T_C(t)
        end for [all time points t]
      end for [all agents stdDev_αA]
    end for [all stdDev_TA(0)]
  calculate average models error ε_avg for all mod-
  els(stdDev_γA, stdDev_TA(0), stdDev_TA(0))
  end for [all stdDev_γA]
calculate average experience level error ε_E for all
experience sequences using ε_avg
end for [all stdDev_E]
```

3.3 Experimental Configurations

In Table 1 the experimental configurations used for the different simulations are summarized. All simulations were run for 500 time steps, and were performed for different values for the agents in the agent-based model to cover different types of populations. The parameter *SS* for the sample of simulation experiments is taken 25: each experiment is run 25 times after which an average is taken. This is meant to undo the randomization effects and to get the general average characteristics of the models. To obtain a wide variety of possible dynamics of the agent-based trust model the agents' initial trust, the agents' flexibility, agents' social influence (in case of indirect experience) and the experience with the trustee were taken exhaustively from a uniform discretised normal distribution with various standard deviations.

Table 1. Experimental configurations

Name	Symbol	Value
Total time steps	*TT*	*500*
Number of agents	*N*	*10, 20, 30, 40, 50*
Samples of simulation experiments	*SS*	*25*
Standard deviation and mean for direct experience	$stdDev_E$, $mean_E$	*0.00-0.24, 0.50*
Standard deviation and mean for rate of change	$stdDev_\gamma$, $mean_\gamma$	*0.00-0.24, 0.50*
Standard deviation and mean for initial trust	$stdDev_{T(0)}$,$mean_{T(0)}$	*0.00-0.24, 0.50*
Standard deviation and mean for social influence	$stdDev_\alpha$ $mean_\alpha$	*0.00-0.24, 0.50*

Given the above experimental configurations, time complexity for the simulation experiments for the ALGORITHM *S1* and *S2* are following

$$O(S1) = O(stdDev_E \,.\, stdDev_\gamma \,.\, stdDev_{TA(0)} \,.\, TT \,.\, N \,.\, SS) \tag{24}$$

$$O(S2) = O(stdDev_E \,.\, stdDev_\gamma \,.\, stdDev_{TA(0)} \,.\, stdDev_\alpha \,.\, TT \,.\, N \,.\, SS) \tag{25}$$

For $stdDev_E$, $stdDev_\gamma$, $stdDev_{TA(0)}$ and $stdDev_\alpha$ ranging from *0.00* to *0.24*, 500 time steps for simulation, 50 agents and 25 samples of simulation the approximate number for the instruction count for ALGORITHMS *S1* and *S2* becomes 9.76×10^9 and 1.22×10^{13} respectively.

3.4 Simulation Results

The algorithm *S1* and *S2* specified in Section 3.1 and 3.2 was implemented in the C++ programming language to conduct the simulations experiments using the configuration as described in Table 1, and to compare the agent-based and population-based trust models. In this section some of the simulation results are discussed

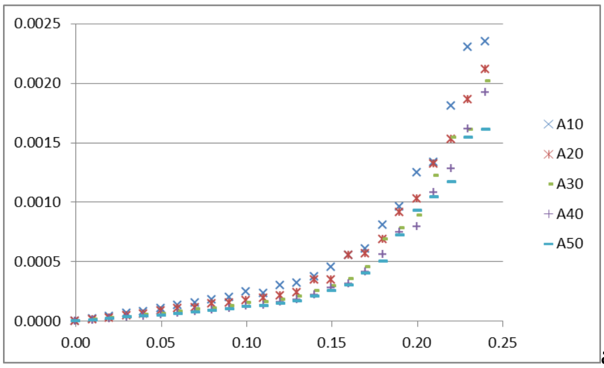
3.4.1 Comparison of Trust Models Incorporating Direct Experiences

In this section results of two experiments are presented for the comparison of trust models incorporating direct experiences only.

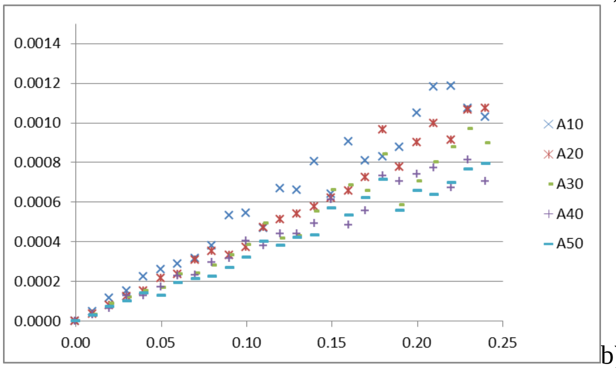
3.4.1.1 Experiment 1 - Variation in the values of the agents' initial trust and trust flexibility parameter

In this experiment the agents' initial trust and flexibility parameter were taken from a discretised uniform normal distribution with mean *0.50* and standard deviation varying from *0.00* to *0.24*. Here it is assumed that the trustee provides a constant experience value *0.50* at all-time points to each agent in the agent-based model and the population-based model. In order to see the effect of the population size on this experiment, this experiment was executed for different numbers of agents, varying from 10 to 50. Some of the results are shown in the following graphs. In Fig. 3a the horizontal axis represents standard deviation of γ_A in the agent-based trust model, varying from *0.00* to *0.24*, and the vertical axis shows the average error between the agent-based and population-based models for all standard deviations of $T_A(0)$ varying from *0.00* to *0.24* against each standard deviation of γ_A. In Fig. 3b the horizontal axis represents the standard deviation of $T_A(0)$ of the agent-based trust model, varying from *0.00* to *0.24* and the vertical axis shows the average error of all standard deviations of $T_A(0)$ varying from *0.00* to *0.24* for each standard deviation of $T_A(0)$.

From these graphs it can be seen that the error between the agent-based and population-based trust model increases when $T_A(0)$ or γ_A of the agent-based model are taken from a higher standard deviation value. Also a higher number of agents in the agent-based model shows less deviation from the population-based model. Furthermore, the higher peak in the left-hand graph and the higher fluctuation in the right-hand graph show that the agent-based model is much more sensitive to variation in γ_A as compared to $T_A(0)$.



a)



b)

Fig. 3. a) Effect of variation in γ (stdDev$_{\gamma A}$ vs. ε_{avg})
b) effect of variation in T(0) (stdDev$_{TA(0)}$ vs. ε_{avg})

3.4.1.2 Experiment 2 - Variation in the experience value from the trustee

This experiment is conducted in two parts; a) multiple experiment are conducted where trustee gives a different constant experience and then output of the models is compared, b) and an exhaustive simulation of experience values given by the trustee from a discretised uniform normal distribution with standard deviation stdDev_E which varies in interval *0.00* to *0.24* around the mean value *0.50* are used.

a) Case 1: In this experiment the agents' initial values for trust and the flexibility parameter were taken from a discretised uniform normal distribution with mean *0.50* and standard deviation varying from *0.00* to *0.24*. Here it was assumed that the trustee provides constant experience value *0.00*, *0.25*, *0.50*, *0.75*, and *1.00* at all time points to each agent in the agent-based model and the population-based model in five different experiments. Also in order to see the effect of the population size on this experiment, this experiment was executed for different numbers of agents varying from 10 to 50. Some of the results are shown in Fig. 4; here the horizontal axis shows the number of agents in the agent-based model while the vertical axis represents the error ε_{avg} for all models as described in the previous section.

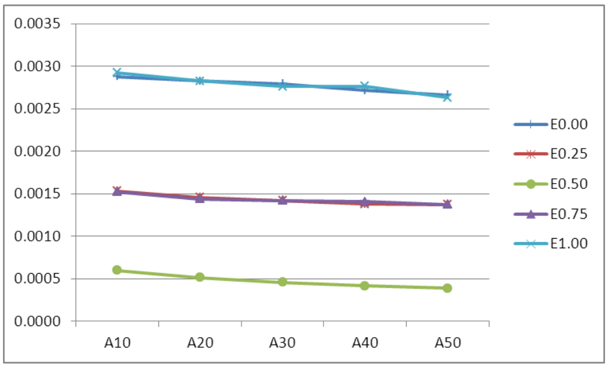


Fig. 4. Effect of different experience values
(Number of agents vs. ε_{avg} for different experience values)

Here it can be seen that upon increase in deviation of the experience value from *0.50* (upward or downward), the agent-based trust model shows a higher error in comparison with the population-based model. It can also be noted that with an increase in the number of agents in the agent-based model, the population-based model approximates the agent-based model more accurately, with lower error value.

b) Case 2: In this experiment an exhaustive simulation was performed where the trustee gives experience values from a discretised uniform normal distribution with standard deviation $stdDev_E$ which is from the interval *0.00* to *0.24* and around the mean value *0.50*. For each value of $stdDev_E$ the agents' initial trust and flexibility parameter were taken from a discretised uniform normal distribution with mean value *0.50* and standard deviation varying from *0.00* to *0.24* (see ALGORITHM S1). Also in order to see the effect of the population size on this experiment, the experiment was executed for different numbers of agents varying from 10 to 50. Some of the results are shown in Fig. 5; here the horizontal axis shows the number of agents in the agent-based model, while the vertical axis represents the average ε_{avg} of errors for a set of models where the trustee gives experience values with standard deviation $stdDev_E$. In Fig. 5a, for example, the curve *E(0.00-0.04)* represents the average error for five models when the trustee gave experiences with standard deviation of *0.00*, *0.01*, *0.02*, *0.03* and *0.04* around mean of *0.50*. These values were averaged and shown here for the sake of presentation. Here it can be seen that for lower deviation in experience value, larger numbers of agents produce a lower error in comparison to smaller number of agents in the agent-based model, while when the trustee gives experience values with a higher standard deviation, then higher numbers of agents show a higher error as compared to lower numbers of agents in the agent-based model. In Fig. 5b the horizontal axis represents the standard deviation in the experience values *E* given by the trustee, varying from *0.00* to *0.24* and the vertical axis shows the average error of all models with standard deviations of the agent attributes $\gamma_A(0)$ and $T_A(0)$ in the agent-based model, varying from *0.00* to *0.24*. Here it can be seen that upon an increase in standard deviation of experience value given by the trustee, the average error between the agent-based and population-based model increases for all population sizes. In Fig. 5c the horizontal axis shows the number of agents in the agent-based model while the vertical axis represents the average errors ε_{avg} for all models, where the trustee gives experience values with standard deviation $stdDev_E$ (varying from *0.00* to *0.24*), the agents in the agent-based model have attributes γ_A and $T_A(0)$ with standard deviations $stdDev_{\gamma A}$ and $stdDev_{TA(0)}$ (varying from *0.00* to *0.24*). Here it can be observed that the population-based trust model provides a more accurate approximation of the agent-based model, when having larger numbers with an exception of an agent-based model with very small numbers (10 agents).

In all these experiments it is observed that the maximum root mean squared error between agent-based and population-based trust model does not exceed *0.017959*, which means that this population-based trust model is a very accurate representation of the agent-based model.

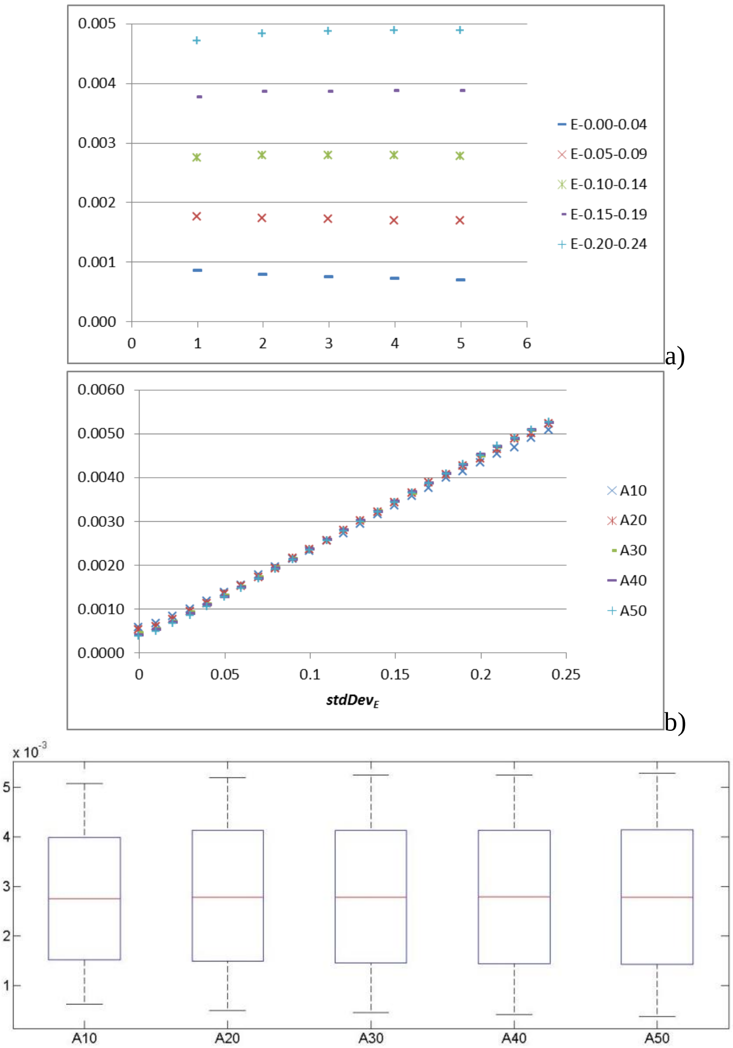


Fig. 5. a) Effect of change in experience for $stdDev_E$ in range *0.00 – 0.24* (number of agents vs. ε_{avg} for different experience values), b) Difference between agent-based and population-based trust models upon variation in experience values ($stdDev_E$ vs. ε_{avg}), c) Average difference (error) between the agent-based and population-based trust models for all possible standard deviations of $stdDev_\gamma$, $stdDev_{TA(0)}$ and $stdDev_E$ (number of agents vs. *average* ε_{avg})

3.4.1.3 Computational time complexity of experiments

The computation complexities of the experiments described above are primarily based on the complexity estimations presented in Section 3.4. It can be observed that that the nature of experiments was exhaustive, hence to conduct them on a desktop PC would require a large amount of time. So these experiments were conducted on the

Distributed ASCI Supercomputer version 4, DAS-4 [3]. The computation complexity of these experiments are shown in Fig. 5. In this figure the horizontal axis represents the different experiments described in the previous sections and the vertical axis shows the number of hours required to complete these experiments on a PC and on DAS-4. Here it should be noted that for these experiment 25 nodes of DAS-4 were utilized. As it can be seen from Fig. 6 all three experiments were expected to take approximately 6.41 hours on a single machine while usage of DAS-4 has reduced this time to approximately 0.25 hours.

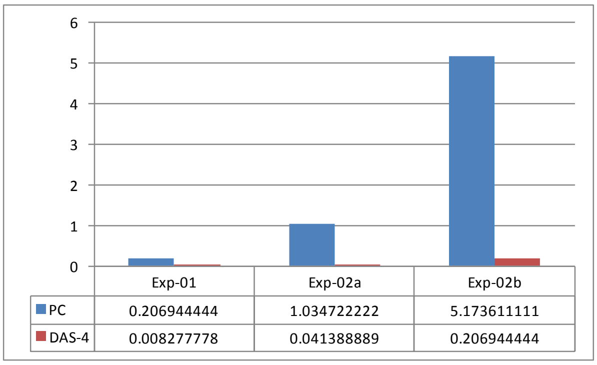


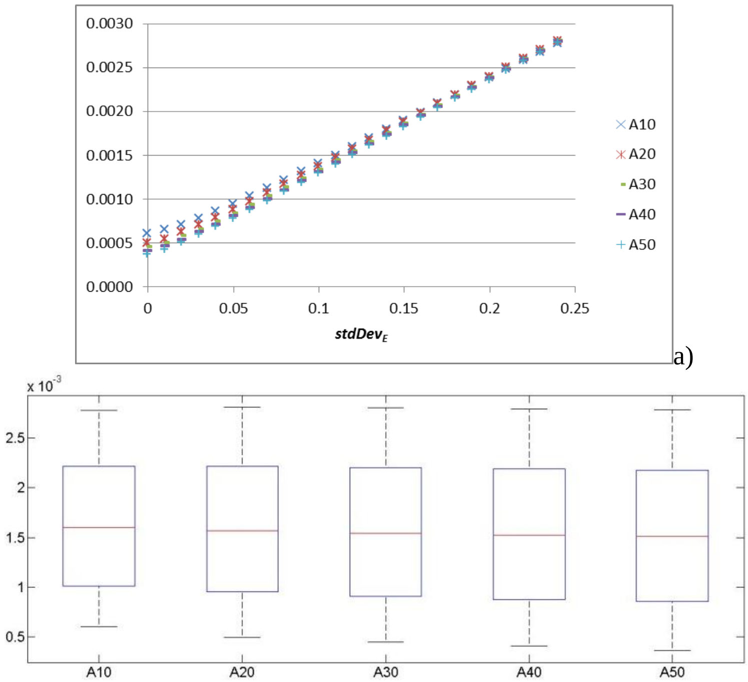
Fig. 6. Total computation time required for the simulation experiments incorporating direct experiences on PC and DAS-4

3.4.2 Comparison of Trust Models Incorporating Indirect Experiences

In this section results of three different experiments are presented for the comparison of trust models incorporating direct and indirect experiences.

3.4.2.1 Experiment 1 - Variation in the experience value from the trustee

In this experiment exhaustive simulations were performed where the trustee gives experience values from a discretised uniform normal distribution around the mean value *0.50* with standard deviation $stdDev_E$ from the interval *0.00* to *0.24*. For each value of $stdDev_E$ the agents' initial trust, flexibility and social influence parameters were taken from a discretised uniform normal distribution with mean value *0.50* and standard deviation varying from *0.00* to *0.24* (see ALGORITHM *S2*). To see the effect of the population size on this experiment, the experiment was executed for different numbers of agents varying from 10 to 50. Some of the results are shown in Fig. 7.



a)

b)

Fig. 7. a) Difference between the two models upon variation in experience values (*$stdDev_E$* vs. *average ε_{avg}* for different number of agents), b) Average difference (error) between the agent-based and population-based trust models for all possible standard deviations of *$stdDev_\gamma$*, *$stdDev_{TA(0)}$*, *$stdDev_\alpha$* and *$stdDev_E$* (number of agents vs. *average ε_{avg}*)

In Fig. 7a the horizontal axis represents the standard deviation in the experience values E given by the trustee, varying from *0.00* to *0.24* and the vertical axis shows the average experience level error ε_E of all models with standard deviations of the agent attributes $T_A(0)$, γ_A and α_A in the agent-based model, varying from *0.00* to *0.24*. Here it can be seen that upon an increase in standard deviation of experience value given by the trustee, the average error between the agent-based and population-based model increases for all population sizes (from about *0.001* to about *0.004*). This error values is lower for higher numbers of agents which shows that the population-based model is a much better approximation of the agent-based based model for higher number of agents. In Fig. 7b the horizontal axis shows the number of agents in the agent-based model while the vertical axis represents the average of the experience level error ε_E for all models, where the trustee gives experience values with standard deviation *$stdDev_E$* (varying from *0.00* to *0.24*), and the agents in the agent-based model have attributes $T_A(0)$, γ_A and α_A with standard deviations *$stdDev_{\gamma A}$*, *$stdDev_{TA(0)}$*, and *$stdDev_\alpha$* (varying from *0.00* to *0.24*). Here it can also be observed that the population-based trust model provides a (slightly) more accurate approximation of the agent-based model, when having larger numbers of agents (from about *0.0026* to about *0.0024*).

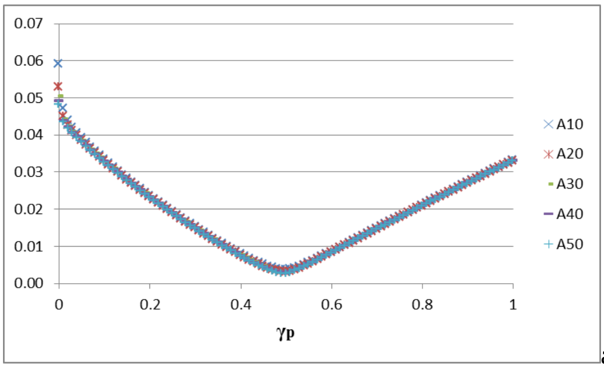
In all these experiments the maximum root mean squared error between agent-based and population-based trust model does not exceed *0.027267*, which means that this population-based trust model is a quite accurate approximation of the agent-based model.

3.4.2.2 Experiment 2 - Exhaustive mirroring of agent-based model into population-based model

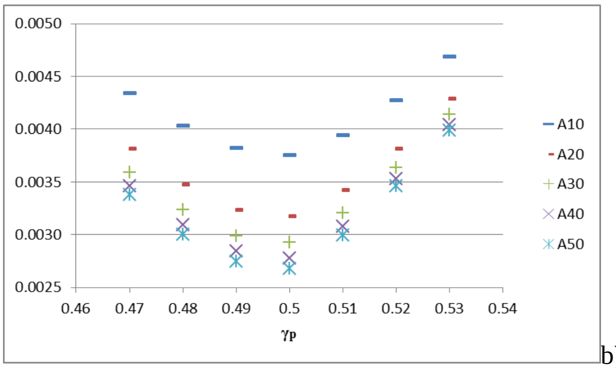
In the previous experiment the attribute values of the population-level model were simply taken as an average of the attribute values of all agents in the agent-level model. However, it cannot be claimed at forehand that this mechanism of abstracting the agent-level model is the most accurate aggregation technique. In order to see whether there is any other instance of the population-level model that can approximate the agent-level models better than the one based on aggregating by averaging, one has to exhaustively simulate all instances of the population-based model against all instances of the agent-based model.

In this experiment such exhaustive simulations were performed, applying a method named as *exhaustive mirroring of models*, adopted from [10]. In this method first a specific trace of the source model for a given set of parameters of source model is generated. Next, the target model is exhaustively (for different parameter settings) simulated to realize this specific trace of the source model. The instance of the target model for specific values of the parameters that generate a minimal error is considered as the best realization of the target model to approximate the source model. This approach is called exhaustive mirroring of one model in the other.

As stated in [10] this mirroring process gives some measure of similarity of the target model against the source model. However, this method of exhaustive mirroring is computationally very expensive. So, for practical reasons in this experiment the population-based model (target) is exhaustively simulated with only one of the three population level parameters, namely the flexibility γ_P of the population-level trust. The other two parameters the population-level (initial trust $T_P(0)$ and social influence α_P) were taken as the average of their counterparts in the agent-level model. Some of the results of this experiment are shown in Fig. 8; In Fig. 8a the horizontal axis represents the exhaustive values for the population-level flexibility parameter γ_P and the vertical axis shows the average experience level error ε_E of all agent-based models with standard deviations of the attributes $T_A(0), \gamma_A, \alpha_A$ and trustee experience $E^d(t)$ varying from *0.00* to *0.24* with mean value *0.5*. Here it can be seen that for lower values of γ_P the average error is much higher and it starts to reduce when γ_P approaches to *0.5* and values of γ_P above *0.5* this error starts to increase. Hence *0.50* is the most accurate representation of γ_P for all agent base models. Further in Fig. 8b same graph is shown in a zoomed-in fashion to show the effect of population size on error value. Here it is seen that larger populations showed lower error than smaller populations.



a)



b)

Fig. 8. a) Difference between agent-based and population-based trust models upon change in population level flexibility parameter γ_P (stdDev$_{\gamma P}$ vs. *average* ε_{avg} for different number of agents), b) Zoomed-in version of Fig. 8a

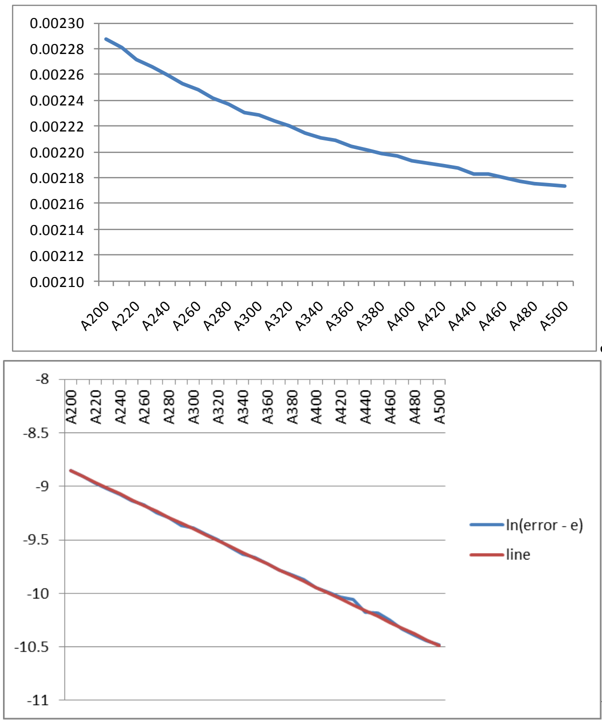
3.4.2.3 Experiment 3: Comparison for larger populations with numbers up to 500 agents

Based on observation from the experiments described above some support was obtained that the value *0.5* for the population-level flexibility parameter γ_P is the most accurate representation of the agent-based model. To get a better impression for the limit value of the error for larger populations, in the next experiment the agent-based model were simulated for larger populations up to *500* agents in size and compared to the population-based model with flexibility parameter $\gamma_P = 0.5$. In this experiment the population was varied from *200* to *500* agents with an increment of *10* agents per population size. Experimental configurations in this experiment were taken from Table 1. Results are shown in Fig. 9; In Fig. 9a the horizontal axis represents the different population sizes varying from 200 to 500 agents and the vertical axis shows the average difference between agent and population level models. Here it can be seen that on an increase in number of agents in population base model difference between models decreases from about *0.00229* (for *200* agents) to about *0.00218* (for *500* agents). It has been analysed in how far the approximation of the limit value for the error for larger populations is exponential and how the limit value can be estimated from the obtained trend. To this end Fig. 9b depicts for a certain value of *e* (an assumed limit value) the graph of the logarithm of the distance of the error to *e*, expressed as *ln(error – e)*. This graph (in blue) is compared to a straight line (in red). It turns out that in 6 decimals the straight line is approximated best for limit value *e = 0.002145*, and the approximation of this limit value for *e* goes exponentially according to an average (geometric mean) factor *0.947149* per increase of *10* agents.

In summary, given that the error found for *N = 200* is *0.002288*, based on this extrapolation method the difference between the agent-based and population-based model for larger population sizes $N \geq 200$ can be estimated as

$$\text{est\_error}(N) = 0.002145 + (0.002288-0.002145)*0.947149^{N-200}$$

$$= 0.002145 + 0.000143*0.947149^{N-200} \qquad (26)$$



a)

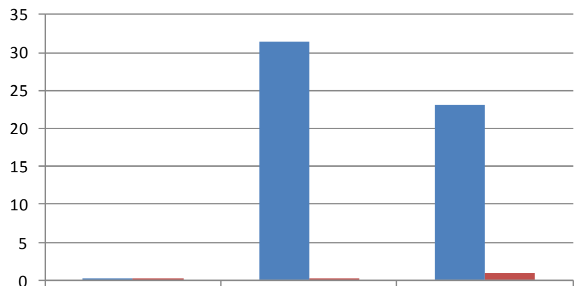
b)

Fig. 9. a) Difference between agent-based and population-based trust models upon change in population size on level flexibility parameter γ_P *= 0.5* (number of agents vs. *average* ε_{avg}), b) Graph of *ln(error – e)* compared to a straight line for *e = 0.002145* (number of agents vs. ε)

This estimation predicts that always an error of at least *0.002145* is to be expected; this actually is quite low, but it will not become still lower in the limit for very large *N*. It turns out that the difference between actual error and estimated error using the above formula in equation 26, for all *N* between *200* and *500* is less than 2.10^{-6}, with an average of 7.10^{-7}. Note that by having this estimation of the error, it can also be used to correct the population-based model for it, thus in a cheap manner approximating the agent-based model by an accuracy around 10^{-6}.

3.4.2.4 Computational time complexity of experiments

The computation complexities of the experiments described above are primarily based on the complexity estimations presented in Section 3.4. It was clear that the nature of these experiments was exhaustive, hence to conduct them on a desktop PC would require a large amount of time. So these experiments were conducted on the Distributed ASCI Supercomputer version 4, DAS-4 [3]. The computation complexity of these experiments is shown in Fig.10. In this figure the horizontal axis represents the different experiments described in the previous sections and the vertical axis shows the number of days required to complete these experiments on a PC and on DAS-4. Here it should be noted that for experiment 1 and 3, 25 nodes of DAS-4 are used while for experiment 2, 100 nodes of DAS-4 were utilized. As it can be seen from Figure 5 all three experiments were expected to take approximately 55 days on a single machine while usage of DAS-4 has reduced this time to approximately 1.25 days.



	Exp-01	Exp-02	Exp-03
PC	0.312256944	31.49945602	23.12265046
DAS-4	0.012490278	0.316296296	0.924906019

Fig. 10. Total computation time required for the simulation experiments incorporating direct and indirect experiences on PC and DAS-4

4 Mathematical Analysis

In this section for a number of cases a mathematical analysis is described of how the agent-based and population-based models compare. This analysis is performed in two Sections 4.1 and 4.2 for the trust models incorporating direct experiences and incorporating indirect experiences respectively.

4.1 Trust models incorporating direct experience

A first step is the simple uniform case that all agents have the same flexibility parameter $\gamma_A = \gamma$. Then by summation of the equation (1)

$$T_A(t + \Delta t) = T_A(t) + \gamma * (E(t) - T_A(t)) * \Delta t$$

over all agents (a number *N*) the following is obtained

$$T_C(t + \Delta t) = \frac{1}{N} \Sigma_A T_A(t + \Delta t)$$

$$= \frac{1}{N} \Sigma_A T_A(t) + \frac{1}{N} \Sigma_A \gamma * (E(t) - T_A(t)) * \Delta t$$
$$= \frac{1}{N} \Sigma_A T_A(t) + \gamma * (E(t) - \frac{1}{N} \Sigma_A T_A(t)) * \Delta t$$
$$= T_C(t) + \gamma * (E(t) - T_C(t)) * \Delta t$$

This shows that the notion of collective trust $T_C(t)$ obtained by aggregating the individual trust level in the agent-based model exactly satisfies the equation for the population-based model for $T_P(t)$. So, in this uniform case the population-based model provides exactly the same outcome as the aggregation of the values from the agent-based models.

As shown by the simulations, this perfect match is not the case when the agents have different flexibility characteristics. As an example, suppose the γ_A have deviations ε_A (which are assumed to be at most some ε, and have an average $\frac{1}{N} \Sigma_A \varepsilon_A = 0$) from the aggregated value γ. Then the above calculations can be done in an approximate manner:

$$T_C(t + \Delta t) = \frac{1}{N} \Sigma_A T_A(t + \Delta t)$$

$$= \frac{1}{N} \Sigma_A T_A(t) + \frac{1}{N} \Sigma_A \gamma_A * (E(t) - T_A(t)) * \Delta t$$
$$= \frac{1}{N} \Sigma_A T_A(t) + \frac{1}{N} \Sigma_A (\gamma + \varepsilon_A) * (E(t) - T_A(t)) * \Delta t$$
$$= T_C(t) + \frac{1}{N} \Sigma_A \gamma * (E(t) - T_A(t)) * \Delta t + \frac{1}{N} \Sigma_A \varepsilon_A * (E(t) - T_A(t)) * \Delta t$$
$$= T_C(t) + \gamma * (E(t) - T_C(t)) * \Delta t + \frac{1}{N} \Sigma_A \varepsilon_A * (E(t) - T_A(t)) * \Delta t$$

This introduces an error term per time step. As $\frac{1}{N} \Sigma_A \varepsilon_A = 0$, the error term can be rewritten as follows:

$$\frac{1}{N} \Sigma_A \varepsilon_A * (E(t) - T_A(t)) * \Delta t$$

$$= (\frac{1}{N} \Sigma_A \varepsilon_A * E(t) - \frac{1}{N} \Sigma_A \varepsilon_A T_A(t)) * \Delta t$$
$$= -\frac{1}{N} \Sigma_A \varepsilon_A T_A(t)) * \Delta t$$
$$= \frac{1}{N} \Sigma_A \varepsilon_A (T_C(t) - T_A(t)) * \Delta t$$

This shows that in particular when the agent-based trust values all converge to a certain limit in response to a certain pattern of experiences, the additional error per time step will become very low.

A limit value for $T_A(t)$ (or $T_P(t)$) can be found by an equilibrium analysis: by determining values for which no change occurs. For this case, strictly spoken this means that E is constant, and $\frac{dT}{dt} = 0$, so $T_A = E$. If E is not constant, but has stochastic fluctuations according to some probability density function $pd(E)$ (for example based on a mean value and standard deviation as in Section 3), then still a form of pseudo-equilibrium value for $T_A(t)$ can be determined if instead of E an expectation value $Exp(E)$ for E is taken. Then a (pseudo-)equilibrium value is $T_A = Exp(E)$, where

$$Exp(E) = \int_0^1 E\, pd(E) dE \tag{27}$$

with $pd(E)$ the probability density function for E.

4.2 Mathematical Analysis of Equilibria of the Two Models incorporating agent communication

The agent-based and population-based models can also be analysed mathematically by determining equilibria. These are values for the variables upon which no change occurs anymore. For equilibria also the externally given experience values have to be constant; instead of these values for E also the expectation value for them can be taken. For the population-level model, assuming flexibility $\gamma_P > 0$ an equilibrium has to satisfy $T_P(t) = E_P(t)$ with

$$EP(t) = \alpha P\, T_P\ (t) + (1 - \alpha P) E_P^d(t) \tag{28}$$

Leaving out t, and taking $E = E_P^d$, this provides the following equation in T_P

$$T_P = \alpha_P\, T_P + (1\text{-}\alpha_P)E \tag{29}$$

Thus (assuming $\alpha_P \neq 1$) an equilibrium $T_P = E$ is obtained. In a similar manner for the agent-based model equilibria can be determined. Again, assuming flexibility $\gamma_A > 0$ an equilibrium has to satisfy $T_A(t) = E_A(t)$ for each agent A, this time with

$$E_A(t) = \alpha_A\, E_A^i(t) + (1 - \alpha_A) E_A^d(t) \tag{30}$$

where $E_A^i(t) = \Sigma_{B\neq A}\, T_B(t)/(N\text{-}1)$. This provides N equations

$$\mathrm{T_A(t) = \alpha_A\ \Sigma_{B\neq A}\ T_B(t)/(N\text{-}1) + (1\text{-}\alpha_A)E} \tag{31}$$

By aggregating possible equations from equation 31, and leaving out t, the relation to collective trust can be found:

$\Sigma_A\, T_A/N = \Sigma_A\, [\alpha_A\, \Sigma_{B\neq A}\, T_B/(N\text{-}1) + (1\text{-}\alpha_A)E\ ]/N$

$T_C = \Sigma_A\, \alpha_A\, \Sigma_{B\neq A}\, T_B/(N\text{-}1)N + \Sigma_A\, (1\text{-}\alpha_A)E\ /N$

$= \Sigma_A\, \alpha_A\, [\Sigma_B\, T_B - T_A]/(N\text{-}1)N + (1\text{-}\Sigma_A\, \alpha_A/N)\, E$

$=[\ \Sigma_A\, \alpha_A\, \Sigma_B\, T_B - \Sigma_A\, \alpha_A\, T_A]/(N\text{-}1)N + (1\text{-}\Sigma_A\, \alpha_A/N)\, E$

$=[\ \Sigma_A\, \alpha_A\, T_C\ /(N\text{-}1) - \Sigma_A\, \alpha_A\, T_A\ /(N\text{-}1)N] + (1\text{-}\Sigma_A\, \alpha_A/N)\, E$

$=[(\Sigma_A\, \alpha_A\ /N)\, T_C\, N\ /(N\text{-}1) - \Sigma_A\, \alpha_A\, T_A\ /(N\text{-}1)N] + (1\text{-}\Sigma_A\, \alpha_A/N)\, E\ T_C$

$=[(\Sigma_A\, \alpha_A\ /N)\, T_C + (\Sigma_A\, \alpha_A\ /N)\, T_C/(N\text{-}1)\ - \Sigma_A \alpha_A T_A\ /(N\text{-}1)N] + (1\text{-}\Sigma_A\, \alpha_A/N)\, E$

$= (\Sigma_A\, \alpha_A\ /N)\, T_C + (1\text{-}\Sigma_A\, \alpha_A/N)\, E + [(\Sigma_A \alpha_A/N)\, T_C/(N\text{-}1) - \Sigma_A\, \alpha_A\, T_A\ /(N\text{-}1)N]$

$= (\Sigma_A\, \alpha_A\ /N)\, T_C + (1\text{-}\Sigma_A\, \alpha_A/N)\, E + [(\Sigma_A\, \alpha_A T_C - \Sigma_A\, \alpha_A\, T_A]\ /(N\text{-}1)N$

$$= (\Sigma_A\ \alpha_A /N)\ T_C + (1\text{-}\Sigma_A\ \alpha_A/N)\ E + \ \Sigma_A\ \alpha_A[T_C - T_A] /(N\text{-}1)N$$

So, taking $\alpha_C = \Sigma_A\ \alpha_A /N$ the following equilibrium equation is obtained:

$$(1\text{-}\alpha_C)\ T_C = (1\text{-}\alpha_C)\ E + \Sigma_A\ \alpha_A[T_C - T_A] /(N\text{-}1)N$$

$$T_C = E + \Sigma_A\ \alpha_A[T_C - T_A] /(N\text{-}1)N(1\text{-}\alpha_C)$$

Therefore in general the difference between the equilibrium values for T_C (aggregated agent-based model) and T_P (population-based model) can be estimated as

$$T_C - T_P = T_C - E = \Sigma_A\ \alpha_A[T_C - T_A] /(N\text{-}1)N(1\text{-}\alpha_C)$$

As T_C and T_A are both between 0 and 1, the absolute value of the expression in $T_C - T_A$ can be bounded as follows

$$|\Sigma_A\ \alpha_A[T_C - T_A] /(N\text{-}1)N(1\text{-}\alpha_C)| \leq \Sigma_A\ \alpha_A /(N\text{-}1)N(1\text{-}\alpha_C)| = \alpha_C/(N\text{-}1)(1\text{-}\alpha_C)$$

Therefore the following bound for the difference in equilibrium values is found:

$$|T_C - T_P| \ \leq\ \alpha_C /(N\text{-}1)(1\text{-}\alpha_C)$$

This goes to 0 for large N, which would provide the value $T_C = E = T_P$. For $\alpha_C = 0.5$, and $N = 200$, this bound is about 0.005, for $N = 500$, it is about 0.002. These deviations are in the same order of magnitude as the ones found in the simulations. Note that the expression in $T_C - T_A$ also depends on the variation in the population. When all agents have equal characteristics $\alpha_A = \alpha$ it is 0, so that $T_C = E = T_P$.

$$T_C - T_P = \alpha\ \Sigma_A\ [T_C - T_A] /(N\text{-}1)N(1\text{-}\alpha_C) = \alpha\ [\ \Sigma_A\ T_C / N - \Sigma_A\ T_A/ N] /(N\text{-}1)\ (1\text{-}\alpha_C)$$

$$= \alpha\ [T_C - T_C] /(N\text{-}1)(1\text{-}\alpha_C) = 0$$

So also in the case of equal parameter values for α_A it holds $T_C = E = T_P$; note that this is independent of the variation for the other parameters.

5 Conclusion

This paper addressed an exploration of the differences between agent-based and population-based models for trust dynamics, based on both a large variety of simulation experiments and a mathematical analysis of the equilibria of the two types of models.

Trust at an individual agent level considers an agent having trust in a certain trustee. At an agent population level, collective trust considers how much trust for a certain trustee exists in a given population of agents. The dynamics of trust states over time can be modelled per individual in an agent-based manner. These individual trust states can be aggregated to obtain a collective trust state $T_C(t)$ of the population, for example by taking their average. Trust dynamics can be modelled from a population perspective as well, by an equation for a population trust value $T_P(t)$; this is much more efficient computationally. However, at forehand it is not clear in how far this $T_P(t)$ will provide similar results as $T_C(t)$.

In this paper an analysis was reported for both ways of modelling of how close $T_C(t)$ and $T_P(t)$ approximate each other. For trust models incorporating direct experiences only, as an overall result, it was shown that the approximation can be reasonably accurate, and for not too small numbers of agents even quite accurate, with differences of less than 0.02 for trust values in the interval [0, 1].

For the trust models incorporating direct and indirect experiences, it was shown that the differences between the two types of model are quite small, in general below *1%*, and become less for larger numbers of agents. An implication of this is that when for a certain application such an accuracy is acceptable, instead of the computationally more expensive agent-based modelling approach (complexity $O(N^2\tau)$ with N the number of agents and τ the number of time steps), as an approximation also the population-based approach can be used (complexity $O(\tau)$).

The experiments to find these results were conducted on the Distributed ASCI Supercomputer version 4, DAS-4 [3], thereby using 25, resp. 100 processors. The experiments were expected to take approximately 55 days on a single PC; the use of DAS-4 has reduced this time to approximately 1.26 days. In future work more complex trust models can be considered, for example, involving competition or cooperation between multiple trustees, agent’s feeling and biases towards trustee.

Note that modelling trust dynamics in a population-based manner for a population as a whole does not provide ways to derive trust for any partiular individual agent. Only if such an agent is a very average type of agent, its trust may be close to the population’s trust level. In cases that specific subpopulations exist for which trust develops in essentially different ways (for example in opposite ways), all individual trust levels may substantially deviate from the population’s trust level. If such subpopulations can be identified, a more refined population-based approach is possible, where each of the subpopulations is modelled in an independent population-based manner. In this way population-based modelling can be applied across multiple abstraction levels.

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